Pearson Physics Level 30
Unit VIII Atomic Physics: Chapter 15
Solutions

Student Book page 756

Example 15.1 Practice Problems

1. ***Given***

$|\vec{B}| = 2.50\ \text{T}$

$|\vec{E}| = 60\ \text{kN/C} = 6.0\times10^4\ \text{N/C}$

Required

the speed of the beam of electrons (v)

Analysis and Solution

Use the equation $v = \dfrac{|\vec{E}|}{|\vec{B}|}$.

$$v = \frac{6.0\times10^4\ \text{N/C}}{2.50\ \text{T}} = 2.4\times10^4\ \text{m/s}$$

Paraphrase

For the electrons to remain undeflected, they must be travelling with a speed of 2.4×10^4 m/s, perpendicular to both fields.

2. ***Given***

$v_p = 1.0\times10^5$ m/s

$|\vec{B}| = 0.05\ \text{T}$

Required

the magnitude of the electric field $\left(|\vec{E}|\right)$

Analysis and Solution

To find the magnitude of the electric field, use the equation

$$v = \frac{|\vec{E}|}{|\vec{B}|}$$

$$|\vec{E}| = v|\vec{B}| = (1.0\times10^5\ \text{m/s})(0.05\ \text{T}) = 5\times10^3\ \text{N/C}$$

Paraphrase

The protons must be passing through an electric field of magnitude 5×10^3 N/C.

3. ***Given***

$|\vec{E}| = 150\ \text{N/C}$

$v = 75\ \text{km/s} = 7.5\times10^4\ \text{m/s}$

Required
magnitude of the magnetic field strength ($|\vec{B}|$)

Analysis and Solution
To find the magnitude of the magnetic field, use the equation

$$v = \frac{|\vec{E}|}{|\vec{B}|}$$

$$\begin{aligned}|\vec{B}| &= \frac{|\vec{E}|}{v} \\ &= \frac{150 \text{ N/C}}{7.5 \times 10^4 \text{ m/s}} \\ &= 2.0 \times 10^{-3} \ \frac{\text{N} \cdot \text{s}}{\text{mC}} \\ &= 2.0 \times 10^{-3} \text{ T}\end{aligned}$$

Paraphrase
A magnetic field of magnitude 2.0×10^{-3} T will stop ions from being deflected in the electric field.

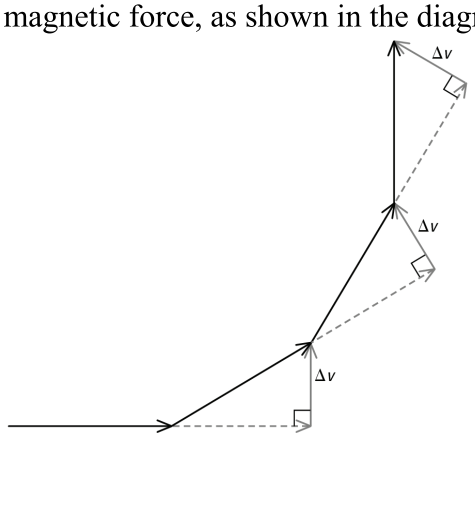
Concept Check

Of the two forces acting on the electrons, it is the magnetic force that depends on velocity, from the equation $|\vec{F}_{\text{m}}| = qv|\vec{B}|$. If speed, *v*, decreases, the magnitude of the magnetic force also decreases. As a result, the electrons experience a greater electric force. Because they are negatively charged, they will begin to accelerate in a direction opposite to the direction of the electric field.

Student Book page 758

Concept Check

The change in velocity, Δ*v*, points inward. If students complete the Δ*v* vectors by subtracting each vector from the previous one (join the vectors tail to tail and then connect the head of the old vector with the head of the new vector), it will be clear that Δ*v* points inward. This Δ*v* arrow represents the direction of acceleration and, therefore, of the magnetic force, as shown in the diagram below.



Example 15.2 Practice Problems

1. *Given*

$r = 1.00 \text{ cm} = 1.00 \times 10^{-2} \text{ m}$
$v = 1.0 \times 10^{6} \text{ m/s}$
$|\vec{B}| = 1.0 \text{ T}$

Required

charge-to-mass ratio for the ion $\left(\frac{q}{m}\right)$

Analysis and Solution

The magnetic force equals the inward (or centripetal) force, so

$$|\vec{F}_{\text{m}}| = |\vec{F}_{\text{c}}|$$

$$|\vec{B}|qv = \frac{mv^2}{r}$$

$$\frac{q}{m} = \frac{v}{|\vec{B}|r}$$

$$\frac{q}{m} = \frac{1.0 \times 10^{6} \text{ m/s}}{(1.0 \text{ T})(1.00 \times 10^{-2} \text{ m})}$$

$$= 1.0 \times 10^{8} \text{ C/kg}$$

Paraphrase

The charge-to-mass ratio for the ion is 1.0×10^{8} C/kg.

2. *Given*

$r = 0.10 \text{ m}$
$|\vec{B}| = 1.0 \times 10^{-4} \text{ T}$

Required

the speed of the electron (v)

Analysis and Solution

From question 1 above,

$$\frac{q}{m} = \frac{v}{|\vec{B}|r}$$

From Example 15.2, $\frac{q}{m} = 1.76 \times 10^{11} \text{ C/kg}$.

$$1.76 \times 10^{11} \text{ C/kg} = \frac{v}{(1.0 \times 10^{-4} \text{ T})(0.10 \text{ m})}$$

$$v = (1.76 \times 10^{11} \text{ C/kg})(1.0 \times 10^{-4} \text{ T})(0.10 \text{ m})$$

$$= 1.8 \times 10^{6} \text{ m/s}$$

Paraphrase

The electron has a speed of 1.8×10^{6} m/s or 1800 km/s.

3. ***Given***

$\frac{q}{m} = 8.04 \times 10^6 \text{ C/kg}$

v = 150 km/s = $150 \times 10^3 \text{ m/s}$

$|\vec{B}|$ = 0.50 T

Required

the radius of the carbon-12 ion's path (r)

Analysis and Solution

From question 1 above,

$$\frac{q}{m} = \frac{v}{|\vec{B}|r}$$

$$r = \frac{v}{|\vec{B}|\left(\frac{q}{m}\right)}$$

$$r = \frac{150 \times 10^3 \text{ m/s}}{(0.50 \text{ T})(8.04 \times 10^6 \text{ C/kg})}$$

$$= 0.037 \text{ m}$$

Paraphrase

The carbon-12 ion travels in a path of radius 0.037 m.

Student Book page 759

Concept Check

A scientific model should provide a plausible description of the phenomenon and also make testable predictions on how the phenomenon will behave during a controlled experiment. The raisin-bun model meets both of these criteria: It predicted the nature of the atom and how charge was distributed in the atom. Both predictions were later disproved by Rutherford's gold-foil experiment. An important criterion of any scientific model is that it makes predictions that could falsify the model.

Student Book page 760

15.1 Check and Reflect

Knowledge

1. (a) ***Given***

$v = 5.0 \times 10^5$ m/s

$|\vec{B}|$ = 100 mT = 100×10^{-3} T

Required

magnitude of the maximum force on the electron (F_{max})

Analysis and Solution

Use the equation $|\vec{F}_m| = qv|\vec{B}|\sin\ \theta$.

The maximum force occurs when $\sin\theta = 1$, which occurs when $\theta = 90°$, so

$$\begin{aligned} F_{\text{max}} &= qv\left|\vec{B}\right|\sin 90° \\ &= (1.60\times10^{-19}\text{ C})(5.0\times10^{5}\text{ m/s})(100\times10^{-3}\text{ T})(1) \\ &= 8.0\times10^{-15}\text{ N} \end{aligned}$$

Paraphrase

The electron experiences a maximum force of magnitude 8.0×10^{-15} N when it enters the magnetic field at right angles ($\theta = 90°$).

(b) ***Given***

$v = 5.0 \times 10^5$ m/s $\quad \left|\vec{B}\right| = 100$ mT

Required

magnitude of the minimum force on the electrons (F_{min})

Analysis and Solution

Use the equation

$$\left|\vec{F}_{\text{m}}\right| = qv\left|\vec{B}\right|\sin\theta$$

The minimum force occurs when $\sin\theta = 0$, which occurs when $\theta = 0°$. Therefore, $F_{\text{min}} = 0$.

Paraphrase

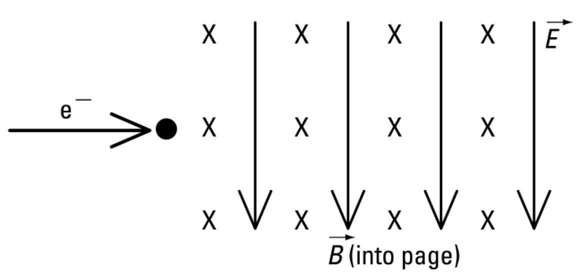
The electron experiences a 0-N force (minimum) when it travels parallel to the magnetic field.

2. The purpose of Thomson's experiment was to measure the effect of the electric field on cathode rays. Without a high vacuum, the electrons leaving the cathode would ionize the surrounding air in the tube. They would lose some of their energy and produce additional charges that would discharge the electric field and obscure the effect of the field on the original electron (cathode ray) emitted by the cathode.

3. Thomson concluded that all cathode rays had identical particles because the particles had the same charge-to-mass ratio, even when different metals were used for the cathodes in the cathode-ray tubes.

Applications

4. (a) Align the particle velocity at right angles to both fields. Arrange the magnetic field so that the magnetic force is in the opposite direction to the electric force. If the electric field points downward, the magnetic field points into the page. All three vectors are mutually perpendicular.



(b) ***Given***

$\left|\vec{E}\right| = 100\text{ kN/C} = 1.00\times10^{5}\text{ N/C}$

$\left|\vec{B}\right| = 0.250\text{ T}$

Required
speed of electrons (v)
Analysis and Solution
Use the equation $v = \frac{|\vec{E}|}{|\vec{B}|}$.

$$v = \frac{1.00 \times 10^5 \text{ N/C}}{0.250 \text{ T}}$$
$$= 4.00 \times 10^5 \text{ m/s}$$

Paraphrase
The electrons should enter the two fields with a speed of 4.00×10^5 m/s.

5. ***Given***
r = 0.040 m
$|\vec{B}|$ = 0.0025 T
Required
speed of electrons (v)
Analysis and Solution
Since the magnetic force acts as the centripetal force,

$$|\vec{F}_{\text{m}}| = |\vec{F}_{\text{c}}|$$
$$|\vec{B}|qv = \frac{mv^2}{r}$$
$$v = \frac{|\vec{B}|qr}{m}$$
$$= \frac{(0.0025 \text{ T})(1.60 \times 10^{-19} \text{ C})(0.040 \text{ m})}{9.11 \times 10^{-31} \text{ kg}}$$
$$= 1.8 \times 10^7 \text{ m/s}$$

Paraphrase
The electrons are moving at a speed of 1.8×10^7 m/s.

6. ***Given***
$v = 1.50 \times 10^5$ m/s
r = 1.00 m
Required
magnitude of the magnetic field ($|\vec{B}|$)
Analysis and Solution
Since the magnetic force acts as the centripetal force,

$$\left|\vec{F}_{\text{m}}\right| = \left|\vec{F}_{\text{c}}\right|$$

$$\left|\vec{B}\right| qv = \frac{mv^2}{r}$$

$$\left|\vec{B}\right| = \frac{mv}{qr}$$

$$= \frac{(1.67 \times 10^{-27}\ \text{kg})(1.50 \times 10^{5}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(1.00\ \text{m})}$$

$$= 1.57 \times 10^{-3}\ \text{T}$$

Paraphrase

A magnetic field of magnitude 1.57×10^{-3} T is needed to deflect the beam of protons.

7. Using the right-hand rule for positive charge motion, the thumb points in the direction of positive charge flow—in the plane of the page counterclockwise, therefore left, and fingers point into the page. So, the direction of the magnetic field is into the page.

8. ***Given***

$\left|\vec{E}\right| = 8.00 \times 10^{2}$ N/C

$\left|\vec{B}\right| = 10.0$ mT $= 1.00 \times 10^{-2}$ T

Required

speed of ions (v)

Analysis and Solution

Use the equation $v = \dfrac{\left|\vec{E}\right|}{\left|\vec{B}\right|}$.

$$v = \frac{8.00 \times 10^{2}\ \text{N/C}}{1.00 \times 10^{-2}\ \text{T}}$$

$$= 8.00 \times 10^{4}\ \text{m/s}$$

Paraphrase

The ions will need a speed of 8.00×10^{4} m/s to pass undeflected through the electric and magnetic fields.

9. (a) ***Given***

$q = +1.60 \times 10^{-19}$ C

$v = 1.0 \times 10^{5}$ m/s

$\theta = 90°$

$\left|\vec{E}\right| = 100$ N/C

$\left|\vec{B}\right| = 0.50$ T

Required

net force on the proton $\left(\vec{F}_{\text{net}}\right)$

Analysis and Solution

The net force is the sum of the electric and magnetic forces, where the magnetic force is perpendicular to the electric force.

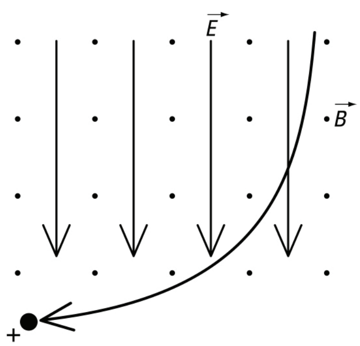
$$\left|\vec{F}_{\text{net}}\right| = \left|\vec{F}_{\text{e}}\right| + \left|\vec{F}_{\text{m}}\right|$$
$$= q\left|\vec{E}\right| + qv\left|\vec{B}\right|\sin 90°$$
$$\left|\vec{F}_{\text{net}}\right| = (1.60\times10^{-19}\text{ C})(100\text{ N/C}) + (1.60\times10^{-19}\text{ C})(1.0\times10^{5}\text{ m/s})(0.50\text{ T})(1)$$
$$= 8.0\times10^{-15}\text{ N}$$

Use the figure to determine the directions of the electric and magnetic forces. Since the charge is positive and the electric field is downward, the electric force also points downward. Use the right-hand rule for a positive charge to determine the direction of the magnetic force. The thumb points to the right, in the direction of charge motion, fingers point out of the page in the direction of the magnetic field, so the palm faces downward, toward the bottom of the page, in the direction of the magnetic force. Since the electric and magnetic forces are both directed downward, $\vec{F}_{\text{net}} = 8.0\times10^{-15}\text{ N [downward]}$.

Paraphrase

The net force on the proton is initially 8.0×10^{-15} N [downward].

(b) The net force will change over time as the proton moves downward because force depends on speed. Speed increases in magnitude as the proton accelerates downward in the electric field. The proton will travel in an arc as it interacts with the magnetic field.



Extensions

10. (a) Use the left-hand rule (for a negative charge). If the thumb points to the right for direction of charge motion and the net force on the negative charge is upward (out of the page), the magnetic field within the detector is toward the bottom of the page.

(b) ***Given***

$q = 5\ \mu\text{C}$

$\left|\vec{B}\right| = 0.05\text{ T}$

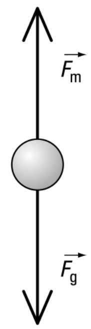
Required

speed (v)

Analysis and Solution

Estimate that the mass of the passenger is 80 kg.

Use the equation $\left|\vec{F}_{\text{m}}\right| = qv\left|\vec{B}\right|\sin\theta$, where $\theta = 90°$. When the passenger is weightless, $\left|\vec{F}_{\text{m}}\right| = \left|\vec{F}_{\text{g}}\right|$, where $\left|\vec{F}_{\text{g}}\right| = mg$.



$$q v|\vec{B}| = mg$$

$$v = \frac{mg}{q|\vec{B}|}$$

$$= \frac{(80 \text{ kg})(9.81 \text{ m/s}^2)}{(5 \times 10^{-6} \text{ C})(0.05 \text{ T})}$$

$$= 3 \times 10^9 \text{ m/s}$$

Paraphrase

In order to feel weightless, the passenger would need to travel faster than the speed of light, which is impossible.

(c) Using an airport metal detector is, therefore, not a practical way to achieve weightlessness. In fact, the physics we use would break down much before $v = 3 \times 10^8$ m/s (the speed of light). We need to use Einstein's theory of relativity to answer this question adequately.

Student Book page 763

Concept Check

The electron's mass is extremely small. It could not, therefore, be measured using the gravitational force and its interaction with other bodies. The charge-to-mass ratio is a much simpler way to determine the electron's mass, providing the charge is known.

Example 15.3 Practice Problems

1. ***Given***

$m = 2.4 \times 10^{-14}$ kg

$\vec{E} = 5.0 \times 10^5$ N/C [up]

Required

number of electrons gained or lost by the sphere (q)

Analysis and Solution

If the charge is suspended, $|\vec{F}_g| = |\vec{F}_e|$ and $\vec{F}_e$ must be directed upward to balance $\vec{F}_g$.

Since the electric force is also upward, the charge must be positive, so it must have lost electrons.

$mg = \left|\vec{E}\right| q$

$$q = \frac{mg}{\left|\vec{E}\right|}$$

$$= \frac{\left(2.4 \times 10^{-14}\ \text{kg}\right)\left(9.81\ \text{m/s}^2\right)}{5.0 \times 10^5\ \text{N/C}}$$

$$= \frac{4.7 \times 10^{-19}\ \not{C}}{1.60 \times 10^{-19}\ \frac{\not{C}}{\text{e}^-}}$$

$$= 2.9\ \text{e}^-$$

$$= 3\ \text{electrons}$$

Paraphrase

The sphere has lost three electrons.

2. ***Given***

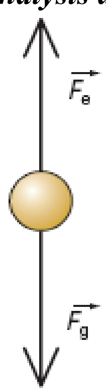
$m = 3.2 \times 10^{-14}$ kg

$q = +2e$

Required

the magnitude and direction of the electric field $\left(\vec{E}\right)$

Analysis and Solution



$\left|\vec{F}_g\right| = \left|\vec{F}_e\right|$, where $\vec{F}_e = q\vec{E}$.

$\vec{F}_e$ is directed upward to balance the gravitational force. Since q is positive, $\vec{E}$ is also directed upward. To find the magnitude of the electric field,

$q\left|\vec{E}\right| = mg$

$$\left|\vec{E}\right| = \frac{mg}{q}$$

$$= \frac{(3.2 \times 10^{-14}\ \text{kg})(9.81\ \text{m/s}^2)}{2\left(1.60 \times 10^{-19}\ \text{C}\right)}$$

$$= 9.8 \times 10^5\ \text{N/C}$$

Paraphrase

The electric field is 9.8×10^5 N/C [up].

Example 15.4 Practice Problems

1. ***Given***

Consider up to be positive.

$m = 2.0 \times 10^{-14}$ kg

$\vec{E} = 1.0 \times 10^{5}$ N/C [up] = $+1.0 \times 10^{5}$ N/C

$q = 5(-1.60 \times 10^{-19}$ C)

$\vec{g} = 9.81$ m/s^2 [down] = -9.81 m/s^2

Required

net force on the sphere ($\vec{F}_{\text{net}}$)

Analysis and Solution

The charge on the sphere is negative, so the electric force is in the opposite direction to the electric field, or down.

Calculate the magnitudes of the electric and gravitational forces acting on the sphere.

$$\begin{aligned}\vec{F}_{\text{g}} &= m\vec{g}\\ &= (2.0\times10^{-14}\ \cancel{\text{kg}})\left(-9.81\ \frac{\text{N}}{\cancel{\text{kg}}}\right)\\ &= -1.96\times10^{-13}\ \text{N}\end{aligned}$$

$$\begin{aligned}\vec{F}_{\text{e}} &= q\vec{E}\\ &= 5(-1.60\times10^{-19}\ \cancel{\text{C}})\left(+1.0\times10^{5}\ \frac{\text{N}}{\cancel{\text{C}}}\right)\\ &= -8.00\times10^{-14}\ \text{N}\end{aligned}$$

The sum of these two forces gives the net force.

$$\begin{aligned}\vec{F}_{\text{net}} &= \vec{F}_{\text{g}} + \vec{F}_{\text{e}}\\ &= -1.96\times10^{-13}\ \text{N} - 8.00\times10^{-14}\ \text{N}\\ &= -2.8\times10^{-13}\ \text{N}\\ &= 2.8\times10^{-13}\ \text{N [down]}\end{aligned}$$

Paraphrase

The net force on the sphere is 2.8×10^{-13} N [down].

2. ***Given***

Consider up to be positive.

$m = 2.0 \times 10^{-14}$ kg

$\vec{E} = 1.0 \times 10^{5}$ N/C [down] = -1.0×10^{5} N/C

$q = 5(-1.60 \times 10^{-19}$ C)

$\vec{g} = 9.81$ m/s^2 [down] = -9.81 m/s^2

Required

acceleration of the sphere ($\vec{a}$)

Analysis and Solution

The gravitational force is the same as in question 1: -1.96×10^{-13} N.

Since the direction of the electric field is reversed, the electric force is in the opposite direction as in question 1, or $+8.00\times10^{-14}$ N.

The sum of these two forces gives the net force.

$$\vec{F}_{\text{net}} = \vec{F}_{\text{g}} + \vec{F}_{\text{e}}$$
$$= -1.96\times10^{-13}\text{ N } + 8.00\times10^{-14}\text{ N}$$
$$= -1.2\times10^{-13}\text{ N}$$

Use the value for net force to determine acceleration:

$$\vec{F}_{\text{net}} = m\vec{a}$$
$$\vec{a} = \frac{\vec{F}_{\text{net}}}{m}$$
$$= \frac{-1.2\times10^{-13}\text{ N}}{2.0\times10^{-14}\text{ kg}}$$
$$= -5.8\text{ m/s}^2$$
$$= 5.8\text{ m/s}^2\text{ [down]}$$

Paraphrase

The acceleration of the sphere is 5.8 m/s^2 [down].

Student Book page 765

15.2 Check and Reflect

Knowledge

1. *Quantization of charge* means that charge on an object can occur only in multiples of some smallest possible amount or quantity. The smallest unit of charge that can be deposited on an object is equal to the charge on a single electron.

2. Millikan was able to determine both the size of the charge on an electron (1.60×10^{-19} C) and its sign (negative).

3. ***Given***

$n = 4$

Required

net charge on oil drop (q)

Analysis and Solution

The drop has *gained* electrons, so it has become more negative.

$q = ne$

$$q = 4(-1.60\times10^{-19}\text{C})$$
$$= -6.40\times10^{-19}\text{C}$$

Paraphrase

The oil drop has a net charge of -6.40×10^{-19} C.

4. ***Given***

Consider up to be positive.

$q = -5e$

$\vec{E}$ = 100 N/C [down] = –100 N/C

Required

electric force on the oil drop $\left(\vec{F}_{\text{e}}\right)$

Analysis and Solution

The charge is negative, so the direction of the electric force is opposite to the direction of the electric field. Therefore, the electric force is acting upward.

To calculate the electric force, use the equation

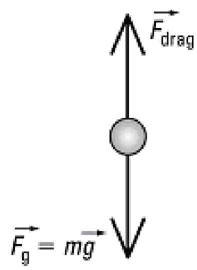
$$\begin{aligned}\vec{F}_e &= q\vec{E} \\ &= (5)\left(-1.60\times10^{-19}\ \cancel{C}\right)\left(-100\ \frac{N}{\cancel{C}}\right) \\ &= +8.00\times10^{-17}\ N\end{aligned}$$

Paraphrase

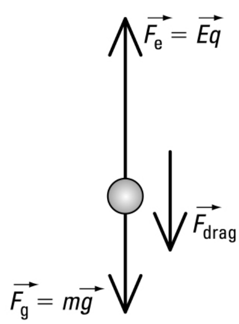
The oil drop experiences a force of 8.00×10^{-17} N [up].

Applications

5. (a) If the oil drop is falling at a constant rate, there is no acceleration and, therefore, the net force is zero. A drag force (air resistance) must be acting, so the oil drop is moving at its terminal velocity.



(b) Initially, the net force is in the upward direction, so the oil drop accelerates upward. From the result in 5(a), eventually, the drag force on the droplet will increase so that the droplet ceases to accelerate and again moves at a constant velocity, this time in the upward direction.



6. (a) ***Given***

$m = 6.9 \times 10^{-15}$ kg

$\vec{E} = 4.23 \times 10^4$ N/C [down]

$a = 0$ m/s^2 (the droplet is motionless)

Required

the charge on the droplet (q)

Analysis and Solution

Since $a = 0$ m/s^2, the electric force balances the gravitational force:

$$|\vec{F}_g| = |\vec{F}_e|$$

$$mg = |\vec{E}|q$$

$$q = \frac{mg}{|\vec{E}|}$$

$$= \frac{(6.9 \times 10^{-15}\ \text{kg})(9.81\ \text{m/s}^2)}{4.23 \times 10^4\ \text{N/C}}$$

$$= 1.6 \times 10^{-18}\ \text{C}$$

The electric force is upward and the electric field is downward, so the droplet has a negative charge.

Paraphrase

The droplet has a charge of -1.6×10^{-18} C or –10e.

(b) $\dfrac{1.6 \times 10^{-18}\ \text{C}}{1.6 \times 10^{-19}\ \text{C/electron}} = 10$ electrons

The droplet has gained 10 electrons.

(c) If the direction of the electric field is suddenly reversed, the electric force would be downward instead of upward, so the droplet would move downward.

Extensions

7. If you look at the differences between the charges, they work out to be the following: 1.8×10^{-19} C, 3.6×10^{-19} C, 5.4×10^{-19} C, etc. They are all multiples of 1.8×10^{-19} C, which shows the expected quantization of charge, but is systematically off by about 12%.

8. Millikan's story is a source of controversy, but historians disagree as to whether Millikan used all of his data. This question could be used as a starting point for a discussion of ethics and honesty in science. A good resource is the book *Betrayers of the Truth* by William Broad and Nicholas Wade. There is no correct answer to this question.

Student Book page 768

Concept Check

Volume varies directly as the cube of physical dimension. The ratio of the radius of the nucleus to the atom's radius is $\dfrac{10^{-14}}{10^{-10}}$ or 10^{-4}. When you cube this number, the ratio becomes 10^{-12}, or roughly one part in a trillion. Your mass occupies one-trillionth of the volume of your body! You are really mostly empty space.

Student Book page 769

Example 15.5 Practice Problems

1. ***Given***

$E_k = 1.6 \times 10^{-12}$ J

$q_1 = q_\alpha = +2\text{e}$ $\qquad$ $q_2 = q_{\text{tin}} = +50\text{e}$

Required
the closest approach (stopping distance) for the alpha particle (d)
Analysis and Solution
Apply the law of conservation of energy. The alpha particle will stop when all of its kinetic energy is converted into potential energy:

$$\begin{aligned}
E_{p_i} + E_{k_i} &= E_{p_f} + E_{k_f} \\
0 + E_{k_i} &= E_{p_f} + 0 \\
E_{k_i} &= E_{p_f} \\
1.6 \times 10^{-12}\ \text{J} &= E_{p_f} \\
E_{p_f} &= \frac{kq_1q_2}{d} \\
d &= \frac{kq_1q_2}{E_{p_f}} \\
&= \frac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(2 \times 1.60 \times 10^{-19}\ \text{C})(50 \times 1.60 \times 10^{-19}\ \text{C})}{1.6 \times 10^{-12}\ \text{J}} \\
&= 1.4 \times 10^{-14}\ \text{m}
\end{aligned}$$

Paraphrase
The alpha particle can come within 1.4×10^{-14} m of the tin nucleus before stopping and being repelled.

2. ***Given***
$d = 5.6 \times 10^{-13}$ m
$q_1 = q_p = +e$
$q_2 = q_{iron} = +56e$
Required
the electric potential energy of the proton (E_p)
Analysis and Solution

$$\begin{aligned}
E_p &= \frac{kq_1q_2}{d} \\
&= \frac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(1 \times 1.60 \times 10^{-19}\ \text{C})(56 \times 1.60 \times 10^{-19}\ \text{C})}{5.6 \times 10^{-13}\ \text{m}} \\
&= 2.3 \times 10^{-14}\ \text{J}
\end{aligned}$$

Paraphrase
A proton located 5.6×10^{-13} m from the centre of an iron nucleus has an electric potential energy of 2.3×10^{-14} J.

Student Book page 770

15.3 Check and Reflect

Knowledge

1. According to Thomson's model, most of the mass and the positive charge in the atom were distributed more or less uniformly throughout the atom. Rutherford's gold-foil experiment gave strong evidence that the positive charge was concentrated in an extremely small volume within the atom. Because the evidence contradicted the prediction, Thomson's model was disproved.

2. In the planetary model of the atom, the negatively charged electrons orbit the positively charged nucleus in a manner similar to the way in which the planets orbit the Sun. Energy changes in the atom are due to energy changes in the radius of electron orbits.

3. (a) ***Given***

$q_1 = q_\alpha = +2\text{e}$

$q_2 = q_{\text{gold}} = +79\text{e}$

$d = 1.0 \times 10^{-10}$ m

Required

the potential energy of the alpha particle (E_p)

Analysis and Solution

$$E_\text{p} = \frac{kq_1q_2}{d}$$

$$= \frac{\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\cancel{\text{C}^2}}\right)(2\times1.60\times10^{-19}\ \cancel{\text{C}})(79\times1.60\times10^{-19}\ \cancel{\text{C}})}{1.0\times10^{-10}\ \cancel{\text{m}}}$$

$$= 3.6\times10^{-16}\ \text{J}$$

Paraphrase

The potential energy of an alpha particle located 1.0×10^{-10} m from the centre of the gold nucleus is 3.6×10^{-16} J.

(b) ***Given***

$q_1 = q_\alpha = +2\text{e}$

$q_2 = q_{\text{gold}} = +79\text{e}$

$d = 1.0 \times 10^{-14}$ m

Required

the potential energy of the alpha particle (E_p)

Analysis and Solution

$$E_\text{p} = \frac{kq_1q_2}{d}$$

$$= \frac{\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^{\cancel{2}}}{\cancel{\text{C}^2}}\right)(2\times1.60\times10^{-19}\ \cancel{\text{C}})(79\times1.60\times10^{-19}\ \cancel{\text{C}})}{1.0\times10^{-14}\ \cancel{\text{m}}}$$

$$= 3.6\times10^{-12}\ \text{J}$$

Paraphrase

The potential energy of an alpha particle located 1.0×10^{-14} m from the centre of the gold nucleus is 3.6×10^{-12} J.

4. In order to get as close as possible to the nucleus, the alpha particle must approach the nucleus head-on. When it is stopped and then repelled, it travels almost straight back, that is, at an angle approaching 180°, which is the maximum possible angle.

Applications

5. Rutherford reasoned that, if both the negative and positive charges were concentrated in the nucleus, then the net charge on the nucleus would be zero (or very low), and the extreme scattering that he observed would not occur. The only way to explain the scattering sometimes observed is for the positive charge to be concentrated in a very small volume within the atom.

6. (a) ***Given***

$V = 1\ \text{m}^3$

$n = 6 \times 10^{28}$

Required

the approximate radius of a gold atom (r)

Analysis and Solution

First consider the volume occupied by each atom.

$$\begin{aligned} V_{\text{atom}} &\approx \frac{1\ \text{m}^3}{6 \times 10^{28}\ \text{atoms}} \\ &\approx 1.7 \times 10^{-29}\ \text{m}^3/\text{atom} \end{aligned}$$

Then use the equation $V = \frac{4}{3}\pi r^3$ to solve for the radius of the atom. (You could also treat each atom as a cube rather than a sphere and obtain nearly the same result.)

$$V = \frac{4}{3}\pi r^3$$

$$\begin{aligned} r &= \sqrt[3]{\frac{3V}{4\pi}} \\ &= 1.6 \times 10^{-10}\ \text{m} \\ &= 2 \times 10^{-10}\ \text{m} \end{aligned}$$

Paraphrase

By using simple arguments, physicists were able to estimate that the approximate radius of a gold atom is 2×10^{-10} m.

(b) Based on these estimates, the gold atom is about 7000 times larger than the gold nucleus $\left(\frac{1.584601442 \times 10^{-10}\ \text{m}}{2.272672 \times 10^{-14}\ \text{m}} = 6972\right)$!

7. (a) Aluminium has a much smaller repelling charge than does gold. As a result, an alpha particle can get much closer to the aluminium nucleus than to the gold nucleus. Rutherford's experiments with aluminium showed that it had a much smaller nucleus than gold and suggested that perhaps alpha particles were able to touch the aluminium nucleus.

(b) ***Given***

$q_1 = q_\alpha = +2\text{e}$

$q_2 = q_{\text{Al}} = +13\text{e}$

$E_\text{k} = 1.2 \times 10^{-12}$ J

Required

radius of an aluminium nucleus, estimate (d)

Analysis and Solution

Apply the law of conservation of energy. When the alpha particle reaches the aluminium nucleus, all of its kinetic energy is converted into potential energy.

$$E_\text{k} = \frac{kq_1q_2}{d}$$

$$E_{p_i} + E_{k_i} = E_{p_f} + E_{k_f}$$
$$0 + E_{k_i} = E_{p_f} + 0$$
$$E_{k_i} = E_{p_f}$$
$$E_{p_f} = \frac{kq_1q_2}{d}$$
$$d = \frac{kq_1q_2}{E_{p_f}}$$
$$= \frac{\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\cancel{\text{C}^2}}\right)(2\times1.60\times10^{-19}\ \cancel{\text{C}})(13\times1.60\times10^{-19}\ \cancel{\text{C}})}{1.2\times10^{-12}\ \text{J}}$$
$$= 5.0\times10^{-15}\ \text{m}$$

Paraphrase

The estimated radius of an aluminium nucleus is 5.0×10^{-15} m .

Extension

8. In 1900, physicists reasoned that, if the positive charges were packed together in the atom's nucleus as tightly as Rutherford's model suggested, the repulsive electrostatic forces between them would be enormous, causing the nucleus to be highly unstable. The stability of the nucleus suggested that there may be a new kind of force that acts within the nucleus to hold the positive charges together. Today, we call this force the strong nuclear force.

Student Book page 774

Concept Check

Draw three concentric circles. Let 1 cm equal the radius of the $n = 1$ energy level. The $n = 2$ level will have a radius of 4 cm and the $n = 3$ level will have a radius of 9 cm. The size of the atom increases with the square of n as n increases.

Student Book page 775

Concept Check

Yes. The negative sign indicates that work must be done to remove the electron from the atom. If you add enough energy to the atom to make E_n equal zero, then you have removed the electron from the atom (ionized the atom). Students often find the concept of negative energy puzzling.

Example 15.6 Practice Problems

1. ***Given***

$n_{\text{initial}} = 1$

$n_{\text{final}} = 4$

Required

energy needed to move an electron from the ground state to $n = 4$ (ΔE)

Analysis and Solution

Use either $E_n = -\frac{1}{n^2}\left(\frac{2\pi^2 mk^2e^4}{h^2}\right)$ or $E_n = -\frac{2.18\times10^{-18}\ \text{J}}{n^2}$

$\Delta E = E_4 - E_1$

$$\Delta E = -\frac{2.18\times10^{-18}\ \text{J}}{4^2} - \left(-\frac{2.18\times10^{-18}\ \text{J}}{1^2}\right)$$
$$= \left(-2.18\times10^{-18}\ \text{J}\right)\left(\frac{1}{4^2} - 1\right)$$
$$= 2.04\times10^{-18}\ \text{J}$$

Paraphrase

An electron must gain 2.04×10^{-18} J to make a transition from the ground state to the $n = 4$ energy level in a hydrogen atom.

2. ***Given***

$n_{\text{initial}} = 5$

$n_{\text{final}} = 2$

Required

energy lost by an electron as it drops from energy level $n = 5$ to $n = 2$ (ΔE)

Analysis and Solution

$$E_n = -\frac{E_1}{n^2}$$
$$= -\frac{2.18\times10^{-18}\ \text{J}}{n^2}$$

$\Delta E = E_5 - E_2$

$$\Delta E = -\frac{2.18\times10^{-18}\ \text{J}}{5^2} - \left(-\frac{2.18\times10^{-18}\ \text{J}}{2^2}\right)$$
$$= \left(-2.18\times10^{-18}\ \text{J}\right)\left(\frac{1}{5^2} - \frac{1}{2^2}\right)$$
$$= 4.58\times10^{-19}\ \text{J}$$

Paraphrase

The electron will emit a photon of energy 4.58×10^{-19} J when it jumps from the $n = 5$ to the $n = 2$ energy level.

Student Book page 778

Example 15.7 Practice Problems

1. ***Given***

$n_{\text{initial}} = 5$

$n_{\text{final}} = 2$

Required

wavelength (λ)

Analysis and Solution

Method 1: Use Balmer's formula.

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right)$$

$$\frac{1}{\lambda} = (1.097 \times 10^7\ \text{m}^{-1})\left(\frac{1}{2^2} - \frac{1}{5^2}\right)$$

$$= 2.3037 \times 10^6\ \text{m}^{-1}$$

$$\lambda = \frac{1}{2.3037 \times 10^6\ \text{m}^{-1}}$$

$$= 4.341 \times 10^{-7}\ \text{m}$$

$$= 434.1\ \text{nm}$$

Method 2: Determine the energy lost by the hydrogen atom when its electron drops from a higher to a lower energy level. Then convert this energy to wavelength using Planck's formula, $E = hf$.

$$\Delta E = E_5 - E_2$$

$$= \left(-2.18 \times 10^{-18}\ \text{J}\right)\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right)$$

$$= hf$$

$$= \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E}$$

$$= \frac{\left(6.63 \times 10^{-34}\ \cancel{\text{J}} \cdot \cancel{\text{s}}\right)\left(3.00 \times 10^8\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{\left(2.18 \times 10^{-18}\ \cancel{\text{J}}\right)\left(\frac{1}{2^2} - \frac{1}{5^2}\right)}$$

$$= 4.34 \times 10^{-7}\ \text{m}$$

$$= 434\ \text{nm}$$

Paraphrase

A photon of wavelength 434 nm will be emitted when an electron in a hydrogen atom drops from the $n = 5$ to the $n = 2$ energy level. (The number of significant digits in the answer depends on the method used.)

2. ***Given***

$n_{\text{initial}} = 3$

$n_{\text{final}} = 7$

Required

wavelength (λ)

Analysis and Solution

Method 1: Use Balmer's formula.

$$\frac{1}{\lambda} = R_H\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right)$$

$$\frac{1}{\lambda} = \left(1.097\times10^{7}\ \text{m}^{-1}\right)\left(\frac{1}{7^2} - \frac{1}{3^2}\right)$$
$$= -9.950\times10^{5}\ \text{m}^{-1}$$
$$\lambda = \frac{1}{-9.950\times10^{5}\ \text{m}^{-1}}$$
$$= -1.005\times10^{-6}\ \text{m}$$
$$= -1005\ \text{nm}$$

Method 2:

The photon must have energy equal to

$$\Delta E = \left(-2.18\times10^{-18}\,\text{J}\right)\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right) = \frac{hc}{\lambda}$$
$$\lambda = \frac{hc}{\left(-2.18\times10^{-18}\ \text{J}\right)\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right)}$$
$$= \frac{\left(6.63\times10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{\left(-2.18\times10^{-18}\ \cancel{\text{J}}\right)\left(\frac{1}{7^2} - \frac{1}{3^2}\right)}$$
$$= 1.01\times10^{-6}\,\text{m}$$

Paraphrase

A photon of wavelength 1005 nm (infrared) must be absorbed for the transition from energy level n = 3 to n = 7. (The number of significant digits in the answer depends on the method used.)

Student Book page 779

Concept Check

Both models predict a compact, positively charged nucleus surrounded by electrons. As well, the greater the energy of the electron, the larger is its orbit. A critical difference is the quantization of energy. In the Bohr model, electron orbits have discrete energy values and sizes, whereas in the planetary model, an orbital radius can have any energy value and size.

Student Book page 780

Concept Check

$$\Delta E = E_{\text{final}} - E_{\text{initial}}$$
$$= 1.96\ \text{eV} - 4.17\ \text{eV}$$
$$= -2.21\ \text{eV}$$

The negative sign means that energy is released.

$$\lambda = \frac{hc}{\Delta E}$$
$$= \frac{\left(6.63\times10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{\left(2.21\ \cancel{\text{eV}}\right)\left(1.60\times10^{-19}\ \frac{\cancel{\text{J}}}{\cancel{\text{eV}}}\right)}$$
$$= 5.62\times10^{-7}\ \text{m}$$
$$= 562\ \text{nm} \approx 558\ \text{nm}$$

This wavelength falls within the range of green light.

$$\Delta E = E_{\text{final}} - E_{\text{initial}}$$
$$= 0 - 1.96\ \text{eV}$$
$$= -1.96\ \text{eV}$$
$$\lambda = \frac{hc}{\Delta E}$$
$$= \frac{\left(6.63\times10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{\left(1.96\ \cancel{\text{eV}}\right)\left(1.60\times10^{-19}\ \frac{\cancel{\text{J}}}{\cancel{\text{eV}}}\right)}$$
$$= 6.34\times10^{-7}\ \text{m}$$
$$= 634\ \text{nm}$$

This transition produces red-orange light.

$$\Delta E = E_{\text{final}} - E_{\text{initial}}$$
$$= 0 - 1.90\ \text{eV}$$
$$= -1.90\ \text{eV}$$
$$\lambda = \frac{hc}{\Delta E}$$
$$= \frac{\left(6.63\times10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{\left(1.90\ \cancel{\text{eV}}\right)\left(1.60\times10^{-19}\ \frac{\cancel{\text{J}}}{\cancel{\text{eV}}}\right)}$$
$$= 6.54\times10^{-7}\ \text{m}$$
$$= 654\ \text{nm}$$

This transition produces red light.

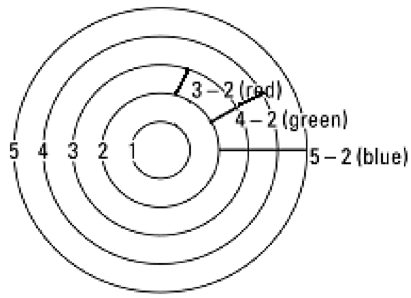
Student Book pages 780–781

15.4 Check and Reflect

Knowledge

1. A quantized process is one that can only occur in discrete multiples of some basic quantity. Energy quantization is an example in which energy can be exchanged (absorbed or emitted) only in multiples of some basic unit or quantum of energy.

2.



3. Bohr's model of the atom predicted the ground-state energy, ionization energy, and the ground-state radius of the hydrogen atom.

4. The differences between energy levels decrease as n increases. Transitions that end in the $n = 3$ energy level are relatively low-energy ones. Electrons can release much more energy by going to the $n = 2$ or $n = 1$ energy levels. Hence, transitions to $n = 3$ produce infrared photons. Transitions to the $n = 1$ energy level emit the most energy, which is in the UV region of the electromagnetic spectrum.

Applications

5. (a) ***Given***

the first four wavelengths on the Balmer series:

$\lambda_4 = 410$ nm

$\lambda_3 = 434$ nm

$\lambda_2 = 486$ nm

$\lambda_1 = 656$ nm

Required

to show that Balmer's formula predicts these wavelengths

Analysis and Solution

Use Balmer's formula, $\frac{1}{\lambda} = R_{\mathrm{H}}\left(\frac{1}{n_{\text{final}}^2} - \frac{1}{n_{\text{initial}}^2}\right)$, to show that n_{initial} is a whole-number integer.

$$\frac{1}{\lambda_1} = R_{\mathrm{H}}\left(\frac{1}{2^2} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$\frac{1}{656\times10^{-9}\ \text{m}} = \left(1.097\times10^{7}\ \text{m}^{-1}\right)\left(\frac{1}{4} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$n = 3$$

$$\frac{1}{\lambda_2} = R_{\mathrm{H}}\left(\frac{1}{2^2} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$\frac{1}{486\times10^{-9}\ \text{m}} = \left(1.097\times10^{7}\ \text{m}^{-1}\right)\left(\frac{1}{4} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$n = 4$$

$$\frac{1}{\lambda_3} = R_H\left(\frac{1}{2^2} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$\frac{1}{434\times10^{-9}\ \text{m}} = \left(1.097\times10^{7}\ \text{m}^{-1}\right)\left(\frac{1}{4} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$n = 5$$

$$\frac{1}{\lambda_4} = R_H\left(\frac{1}{2^2} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$\frac{1}{410\times10^{-9}\ \text{m}} = \left(1.097\times10^{7}\ \text{m}^{-1}\right)\left(\frac{1}{4} - \frac{1}{n_{\text{initial}}^2}\right)$$

$$n = 6$$

Paraphrase

Each value of n is a whole-number integer, so Balmer's formula correctly predicts the wavelengths of photons emitted when a hydrogen atom's electrons drop to the $n = 2$ energy level.

(b) From (a), the wavelength that corresponds to the transition from the $n = 4$ to the $n = 2$ energy level is 486 nm.

(c) ***Given***

$n = 4$

$n = 2$

Required

the energy difference between the $n = 4$ and $n = 2$ states (ΔE)

Analysis and Solution

The energy difference between the energy levels $n = 2$ and $n = 4$ is the same as the energy of the photon. So,

$$\Delta E = hf = \frac{hc}{\lambda}$$

$$\Delta E = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)}{486\times10^{-9}\ \cancel{\text{m}}} = 4.09\times10^{-19}\ \text{J}$$

Paraphrase

The energy of the photon equals the energy difference between the two energy levels, 4.09×10^{-19} J.

6. ***Given***

$\lambda = 633$ nm

Required

energy difference (ΔE)

Analysis and Solution

$$\Delta E = hf = \frac{hc}{\lambda}$$

$$\Delta E = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)}{633\times10^{-9}\ \cancel{\text{m}}}$$

$$= 3.14\times10^{-19}\ \cancel{\text{J}}\times\frac{1\ \text{eV}}{1.60\times10^{-19}\ \cancel{\text{J}}}$$

$$= 1.96\ \text{eV}$$

Paraphrase

The energy difference between these two states is 1.96 eV.

7. (a) The four transitions, from shortest wavelength to longest wavelength, are 4, 2, 1, 3. The shortest wavelengths have the most energy and thus the biggest transitions. Similarly, the longest wavelengths have the least energy and thus the smallest transitions.

(b) Estimates of the energy of each of the transitions are:

1: ΔE = 5.4 eV – 3.5 eV = 1.9 eV
2: ΔE = 3.5 eV – 1.4 eV = 2.1 eV
3: ΔE = 3.5 eV – 2.0 eV = 1.5 eV
4: ΔE = 5.4 eV – 1.5 eV = 3.9 eV

(c) $\Delta E = hf = \frac{hc}{\lambda}$

$$\lambda = \frac{hc}{\Delta E}$$

$$\lambda_1 = \frac{\left(6.63\times10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{1.9\ \cancel{\text{eV}}\times\frac{1.60\times10^{-19}\ \cancel{\text{J}}}{1\ \cancel{\text{eV}}}}$$

$$= 6.54\times10^{-7}\ \text{m}$$

$$= 654\ \text{nm}$$

visible, red

$$\lambda_2 = \frac{\left(6.63\times10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{2.1\ \cancel{\text{eV}}\times\frac{1.60\times10^{-19}\ \cancel{\text{J}}}{1\ \cancel{\text{eV}}}}$$

$$= 5.92\times10^{-7}\ \text{m}$$

$$= 592\ \text{nm}$$

visible, yellow

$$\lambda_3 = \frac{\left(6.63\times10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{1.5\ \cancel{\text{eV}}\times\frac{1.60\times10^{-19}\ \cancel{\text{J}}}{1\ \cancel{\text{eV}}}}$$

$$= 8.29\times10^{-7}\ \text{m}$$

$$= 829\ \text{nm}$$

$$\lambda_4 = \frac{\left(6.63\times10^{-34}\ \cancel{\text{J}}\cdot\cancel{\text{s}}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}\right)}{3.9\ \cancel{\text{eV}}\times\frac{1.60\times10^{-19}\ \cancel{\text{J}}}{1\ \cancel{\text{eV}}}}$$

$$= 3.19\times10^{-7}\ \text{m}$$

$$= 319\ \text{nm}$$

8. The solar spectra show the presence of both sodium and hydrogen atoms, so you know that the Sun is composed of at least these elements. Other spectral features reveal that the Sun consists of almost all the elements that you find on Earth.

9. (a) ***Given***

$n_{\text{initial}} = 2$

$n_{\text{final}} = 3$

Required

the difference in energy (ΔE)

Analysis and Solution

Use the equation $\Delta E = \left(-2.18\times10^{-18}\ \text{J}\right)\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right)$.

$$\Delta E = \left(-2.18\times10^{-18}\ \text{J}\right)\left(\frac{1}{3^2} - \frac{1}{2^2}\right)$$

$$= 3.03\times10^{-19}\ \text{J}$$

Paraphrase

The energy difference between the $n = 2$ and $n = 3$ energy levels is 3.03×10^{-19} J.

(b) ***Given***

$n_{\text{initial}} = 5$

$n_{\text{final}} = 6$

Required

the difference in energy (ΔE)

Analysis and Solution

Use the equation $\Delta E = \left(-2.18\times10^{-18}\ \text{J}\right)\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right)$.

$$\Delta E = \left(-2.18\times10^{-18}\ \text{J}\right)\left(\frac{1}{6^2} - \frac{1}{5^2}\right)$$

$$= 2.66\times10^{-20}\ \text{J}$$

Paraphrase

The energy difference between the $n = 5$ and $n = 6$ energy levels is 2.66×10^{-20} J.

(c) From your answers to (a) and (b), the difference in energy decreases as consecutive energy levels increase. Consider the term $\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right)$. As n increases, the value of this term approaches zero:

$$\frac{1}{n^2}-\frac{1}{(n+1)^2}=\frac{(n+1)^2-n^2}{n^2(n+1)^2}$$
$$=\frac{2n+1}{n^2(n+1)^2}$$

10. Use the expression $\Delta E=\left(-2.18\times10^{-18}\text{ J}\right)\left(\frac{1}{n^2}-\frac{1}{1^2}\right)$. If you let n = 1000, then the $\left(\frac{1}{n^2}-\frac{1}{1^2}\right)$ term differs from 1 by only one part in a million. In other words, the difference term becomes 1 and you are left with $\Delta E=-2.18\times10^{-18}$ J or E_1.

11. (a) ***Given***

n_{initial} = 0

n_{final} = 5

Required

(a) wavelength (λ)

Analysis and Solution

(a) ΔE = 8.85 eV – 0 eV

= 8.85 eV

$$\Delta E=hf=\frac{hc}{\lambda}$$
$$\lambda=\frac{hc}{\Delta E}$$
$$\lambda=\frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{8.85\ \text{eV}\times\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}}$$
$$=1.40\times10^{-7}\text{ m}$$
$$=140\text{ nm}$$

(b) The longest wavelength is emitted by the shortest transition, from n = 6 to n = 5.

ΔE = 9.23 eV – 8.85 eV

= 0.38 eV

$$\Delta E=hf=\frac{hc}{\lambda}$$
$$\lambda=\frac{hc}{\Delta E}$$
$$\lambda=\frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \frac{\text{m}}{\text{s}}\right)}{0.38\ \text{eV}\times\frac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}}$$
$$=3.27\times10^{-6}\text{ m}$$

(c) The number of possible downward transitions that can occur is 5 + 4 + 3 + 2 + 1 = 15

(d) The spectral lines would be close together and similar in colour.

Paraphrase

(a) The wavelength of photon required is 140 nm.

(b) The longest wavelength of photon is 3.27×10^{-6} m.

Extension

12. (a) "Monochromatic" means that the photons produced by the laser are all of the same colour or wavelength. "Coherent" means that the photons are in phase, and "collimated" means that they are all travelling in the same direction.

(b) The spectrum of a laser should be a very sharp emission line.

Student Book page 783

Concept Check

The Bohr model explains that spectral lines are the result of energy quantization and the energy given off or absorbed by electrons as they make a transition from one energy level to another. The splitting of spectral lines suggests that there must be at least one other form of energy quantization that is not accounted for in the Bohr model. As a result, more quantum numbers must be added to the quantum mechanical model of the atom.

Student Book page 784

15.5 Check and Reflect

Knowledge

1. Three failings of the Bohr model of the atom are:
- Although it made use of the concept of energy quantization, the Bohr model did not explain why energy in an atom was quantized or why electrons did not radiate energy as they orbited the nucleus.
- The Bohr model really only worked for the hydrogen atom or for very simple "hydrogenic" atoms or ions.
- The Bohr model could not explain subtle spectral features, such as the splitting of spectral lines in magnetic fields (the Zeeman effect).

2. The Zeeman effect is the splitting of spectral lines in the presence of magnetic fields.

3. An orbital is a pictorial representation of the probability distribution for the location of an electron in an atom. It is not the actual orbit of the electron. In fact, the idea of orbit has no meaning in this context.

Applications

4. (a) ***Given***

$n = 1$

$r_{n=1} = 5.29 \times 10^{-11}$ m

Required

the de Broglie wavelength of the electron (λ)

Analysis and Solution

The de Broglie wavelength must "fit" within the ground-state orbital by forming a standing wave. Use the formula for the circumference of a circle, $\lambda = 2\pi r$, to find the wavelength of the electron.

$$\begin{aligned}\lambda &= 2\pi r \\ &= 2\pi(5.29\times10^{-11}\text{ m}) \\ &= 3.32\times10^{-10}\text{ m}\end{aligned}$$

Paraphrase

The de Broglie wavelength for a ground-state electron in a hydrogen atom is 3.32×10^{-10} m.

(b) ***Given***

$n = 1$

$r_{n=1} = 5.29 \times 10^{-11}$ m

$\lambda = 3.324\times10^{-10}\text{ m}$ from part a

Required

the momentum of the electron (p)

Analysis and Solution

Find the momentum using the equation for de Broglie wavelength, $\lambda = \dfrac{h}{p}$.

$$\begin{aligned}p &= \frac{h}{\lambda} \\ &= \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{3.324\times10^{-10}\text{ m}} \\ &= 1.99\times10^{-24}\text{ kg}\cdot\text{m/s}\end{aligned}$$

Paraphrase

The momentum of the electron is 1.99×10^{-24} kg·m/s.

(c) ***Given***

$n = 1$

$r_{n=1} = 5.29 \times 10^{-11}$ m

$p = 1.995\times10^{-24}\text{ kg}\cdot\text{m/s}$ from part b

Required

kinetic energy of the electron (E_k)

speed of the electron (v)

Analysis and Solution

For kinetic energy, use the equation $E_k = \dfrac{p^2}{2m}$. For speed, use $p = mv$. The mass of an electron is 9.11×10^{-31} kg.

$$\begin{aligned}E_k &= \frac{\left(1.995\times10^{-24}\text{ kg}\cdot\text{m/s}\right)^2}{2\left(9.11\times10^{-31}\text{ kg}\right)} \\ &= 2.18\times10^{-18}\text{ J}\end{aligned}$$

$$\begin{aligned}v &= \frac{p}{m} \\ &= \frac{1.995\times10^{-24}\ \cancel{\text{kg}}\cdot\text{m/s}}{9.11\times10^{-31}\ \cancel{\text{kg}}} \\ &= 2.19\times10^{6}\text{ m/s}\end{aligned}$$

Paraphrase

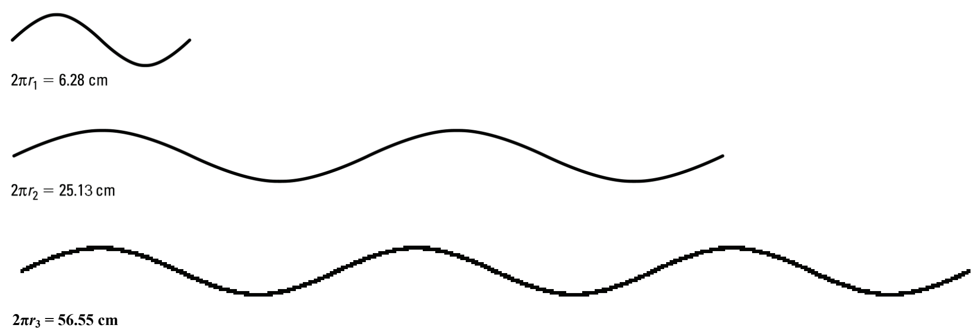
The kinetic energy of the electron is 2.18×10^{-18} J and its speed is 2.19×10^{6} m/s.

5. (a) Use $2\pi r_n = n\lambda_n$ and $r_n = n^2 r_1$.

$$2\pi(r_1 n^2) = n\lambda_n$$
$$\lambda_n = 2\pi n r_1$$
$$= n(2\pi r_1)$$
$$= n\lambda_1$$

Since $\lambda_n = n\lambda_1$, it follows that $\lambda_2 = 2\lambda_1$ and $\lambda_3 = 3\lambda_1$.

(b) The basic relationship is $\lambda_n = n\lambda$. To make a scale diagram, note that $r_n = r_1 n^2$. Let $r_1 = 1$ cm, $r_2 = 4$ cm, and $r_3 = 9$ cm. Fit one wave in $2\pi r_1$, two waves in $2\pi r_2$, and three waves in $2\pi r_3$. A good way to illustrate this relationship is to "unroll" the orbits (see Figure 15.25 in the student book). For example,



Extensions

6. According to Born's interpretation, the meaning of a node in a wave function is zero probability of finding an electron.

7. The transition that produces the 21-cm line is due to a change in alignment between the spin of the electron and the spin of the proton in the hydrogen atom. When the spin axes of the electron and the proton point in the same direction, the hydrogen atom has slightly more energy than when the spin axes are opposed. A hydrogen atom in the excited state spontaneously flips its axes to the lower-energy state and emits the 21-cm radiation line. In Earth's atmosphere, where air density is relatively high, collisions between hydrogen atoms and other atoms are so frequent and so violent that the 21-cm spin-flip does not have a chance to occur. Collisions continuously jostle the electrons and alter the spin states at random. In the high vacuum of space, however, collisions are much less frequent and the spin-flip transition has time to occur.

Student Book pages 786–787

Chapter 15 Review

Knowledge

1. A cathode "ray" is an electron, which carries a negative charge.

2. (a) The electron is negatively charged and will travel in the opposite direction of the field. So, in this case, the force on the electron is up (toward the top of the page).

(b) Use the left-hand rule (for negative charges). The thumb points in the direction of motion, fingers point in the direction of the magnetic field (toward the bottom of the page), and the force on the particle is out of the palm. In this case, the magnetic force is directed out of the page, toward you.

(c) Arrange the magnetic field so that it points into the page. In his case, with the cathode ray moving to the right and the magnetic field into the page, the magnetic force will be directed downward (toward the bottom of the page). If, at the same time, the electric field points downward (toward the bottom of the page), then the electric force will act upward (toward the top of the page). If the velocity of the cathode ray were adjusted so that $\vec{v} = \frac{\vec{E}}{B}$ then the forces would cancel and the net force on the cathode ray would be zero.

3. The beam must be travelling through a magnetic field because it is being bent into a circular path. The bending indicates that the force on the cathode ray is always at right angles to the direction of motion, or velocity, of the ray. This observation is consistent with the effect of a force created by the interaction between the cathode ray and a magnetic field.

4. Thomson knew neither the mass nor the charge of an electron, but he was able to measure the speed of the cathode rays in crossed electric and magnetic fields. He measured the deflection of the rays with only the magnetic field switched on. From his measurements, he was able to derive an expression for the electron's charge-to-mass ratio.

5. By using the charge of the electron (q), determined by Millikan's experiment, and Thomson's charge-to-mass ratio $\left(\frac{q}{m}\right)$, physicists could solve for charge and thus determine the mass of the electron.

6. Since the dust particle has lost electrons, it has become positively charged. Its charge is now 23 × (+1.60 × 10^{-19} C) = +3.68 × 10^{-18} C.

7. ***Given***

m = 1.00 kg of electrons

Required

the charge carried by 1.00 kg of electrons (q)

Analysis and Solution

Determine the number of electrons in 1.00 kg, where the mass of one electron is 9.11 × 10^{-31} kg.

$$n = \frac{1.00\ \cancel{\text{kg}}}{9.11 \times 10^{-31}\ \cancel{\text{kg}}/\text{electron}}$$
$$= 1.098 \times 10^{30}\ \text{electrons}$$

Multiply the number of electrons by –1.60 × 10^{-19} C, the charge on one electron.

$$q = (1.098 \times 10^{30})(-1.60 \times 10^{-19}\ \text{C})$$
$$= -1.76 \times 10^{11}\ \text{C}$$

Paraphrase

One kilogram of electrons has a charge of –1.76 × 10^{11} C.

8. An alpha particle is a helium nucleus—a particle consisting of two protons and two neutrons.

9. In Rutherford's experiment, alpha particles were shot at a thin gold foil. Most of the time, the alpha particles travelled straight through the foil, but occasionally, they ricocheted or scattered backwards at different angles. Hence, Rutherford's gold-foil experiment is sometimes called a "scattering experiment".

10. Thomson's model predicted that most of the mass and the positive charge in the atom were distributed more or less uniformly throughout the atom. It predicted mild scattering of alpha particles as they passed by (and through) the atoms in the gold foil. Thomson's model could not explain the very extreme scattering observed in some cases, and was therefore inconsistent with the results of Rutherford's gold-foil experiment.

11. (a) An emission line spectrum is a bright-line spectrum produced when a hot gas emits energy in the form of photons of light.

(b) The easiest way to produce an emission line spectrum is to heat a gas. Heating excites the electrons in the atoms of the gas to higher energy levels from which they make downward jumps (transitions) and emit photons.

12. Fraunhofer lines are the faint, dark absorption lines of elements that appear in the solar spectrum.

13. Emission lines occur at distinct wavelengths. Since the energy of a photon is related to its wavelength, emission lines signify that distinct amounts of energy are being given off by the atom. This effect demonstrates Planck's concept of energy quantization.

14. The ground state is the lowest possible energy state that an atom can have. An excited state is any other energy state in the atom. An atom must gain energy in order to move from the ground state to an excited state.

15. (a) Any transition from a lower to a higher state implies that the atom has gained energy. The transitions that represent an atom gaining energy are $n_i = 1$ to $n_f = 5$ and $n_i = 2$ to $n_f = 5$.

(b) The atom gains the most energy from the $n_i = 1 \rightarrow n_f = 5$ transition.

(c) Transition $n_i = 4$ to $n_f = 3$ would release the least amount of energy because it represents the smallest drop in energy level. This transition would therefore emit the longest wavelength photon.

16. ***Given***

$n = 3$

Required

radius of the hydrogen atom at $n = 3$ (r_3)

Analysis and Solution

Use the equation $r_n = r_1 n^2$, where $r_1 = 5.29 \times 10^{-11}\ \text{m}$.

$$r_3 = \left(5.29 \times 10^{-11}\ \text{m}\right)3^2$$
$$= 4.76 \times 10^{-10}\ \text{m}$$

Paraphrase

The radius of the hydrogen atom in the $n = 3$ state is $4.76 \times 10^{-10}\ \text{m}$.

17. In the Bohr model, electrons orbit the nucleus in paths that are determined by the electrical interaction between the negatively charged electron and the positively charged nucleus. In the quantum model, electrons do not orbit. Instead, the orbital is a mathematical depiction of the probability that an electron will be found in a given location.

18. Unlike both the Rutherford and Bohr models of the atom, electrons in the quantum model are not travelling in circular, hence accelerating, orbits. They do not violate

Maxwell's idea that accelerating charges radiate because, according to the quantum model, they are not accelerating. What electrons are doing is strictly indeterminate until they make an energy transition or interact in some way.

Applications

19. *Given*

$v_{\perp}$ = 1.0 km/s = 1.0 × 10^3 m/s

$|\vec{B}|$ = 1.5 T

Required

the magnitude of the magnetic force on the electron ($|\vec{F}_{\mathrm{m}}|$)

Analysis and Solution

Use the equation $|\vec{F}_{\mathrm{m}}| = qv|\vec{B}|\sin\ \theta$.

Since v is perpendicular to $|\vec{B}|$, θ = 90°, and sin 90° = 1.

$$|\vec{F}_{\mathrm{m}}| = (1.60 \times 10^{-19}\ \mathrm{C})(1.0 \times 10^{3}\ \mathrm{m/s})(1.5\ \mathrm{T})(1)$$
$$= 2.4 \times 10^{-16}\ \mathrm{N}$$

Paraphrase

The electron will experience a force of 2.4 × 10^{-16} N at right angles to both its velocity and the magnetic field.

20. (a) *Given*

$n = 2$

Required

the speed of an electron (v)

Analysis and Solution

Determine the kinetic energy of the electron by using the law of conservation of energy. Then use the equation for kinetic energy to find the speed of the electron in the $n = 2$ energy level.

To find the energy of the electron in the $n = 2$ energy level, use the equation $E_n = -\dfrac{2.18 \times 10^{-18}\ \mathrm{J}}{n^2}$.

$$E_2 = -\frac{2.18 \times 10^{-18}\ \mathrm{J}}{2^2}$$
$$= -5.45 \times 10^{-19}\ \mathrm{J}$$

For the radius of the $n = 2$ energy level,

$r_n = r_1 n^2$, where $r_1 = 5.29 \times 10^{-11}$ m

$$r_2 = (5.29 \times 10^{-11}\ \mathrm{m})(2^2)$$
$$= 2.116 \times 10^{-10}\ \mathrm{m}$$

To find the potential energy of the electron with respect to the proton nucleus of the hydrogen atom, use the equation for electric potential energy, $E_{\mathrm{p}} = \dfrac{kq_1q_2}{d}$.

$$E_p = \frac{kq_1q_2}{r_2}$$
$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\ \text{C})(+1.60\times10^{-19}\ \text{C})}{2.116\times10^{-10}\ \text{m}}$$
$$= -1.088\times10^{-18}\ \text{J}$$

Since $E_k + E_p = E_n$, calculate E_k using this equation.

$$E_k = E_2 - E_p$$
$$= -5.45\times10^{-19}\ \text{J} - (-1.088\times10^{-18}\ \text{J})$$
$$= 5.426\times10^{-19}\ \text{J}$$

Find v using $E_k = \frac{1}{2}mv^2$.

$$v = \sqrt{\frac{2E_k}{m}}$$
$$= \sqrt{\frac{2(5.426\times10^{-19}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}}$$
$$= 1.09\times10^6\ \text{m/s}$$

Paraphrase

The speed of an electron in the n = 2 energy level of the Bohr model of the hydrogen atom is 1.09×10^6 m/s.

(b) The Bohr model has some characteristics of classical physics, so it is possible to make a clear distinction between kinetic and potential energy within the Bohr model of the atom. In contrast, the quantum model does not include the idea of orbital motion. Instead, it gives the energy of an electron in a particular energy level. Also, the quantum model does not specify an exact location for the electron—it only gives probabilities for its location. Since we do not know an electron's exact location, we cannot calculate an exact value for its potential energy using the quantum model and, therefore, we cannot determine its speed.

21. ***Given***

$m = 2.0 \times 10^{-15}$ kg

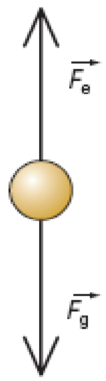
$q = +3e = +3(1.60 \times 10^{-19}\ \text{C})$

Required

the electric field needed to suspend the oil drop $(\vec{E})$

Analysis and Solution

The oil drop is suspended, so the electrical and gravitational forces balance each other. The FBD for this situation is:



From the FBD,

$$\left|\vec{F}_e\right| = \left|\vec{F}_g\right|$$

$$\left|\vec{E}\right| q = mg$$

$$\left|\vec{E}\right| = \frac{mg}{q}$$

$$\left|\vec{E}\right| = \frac{(2.0 \times 10^{-15}\ \text{kg})(9.81\ \text{m/s}^2)}{3(1.60 \times 10^{-19}\ \text{C})}$$

$$= 4.1 \times 10^4\ \text{N/C}$$

The charge is positive, so the electric field is in the same direction as the electric force: up.

Paraphrase

An electric field of 4.1×10^4 N/C [up] will be required to suspend the oil droplet.

22. (a) ***Given***

$n = 3$

Required

change in energy for the first three Paschen transitions (ΔE)

Analysis and Solution

From Figure 15.21 in the student text, the Paschen transitions are between energy levels 4 and 3, 5 and 3, and 6 and 3.

$$E_n = E_1\left(\frac{1}{n^2_{\text{final}}} - \frac{1}{n^2_{\text{initial}}}\right), \text{ where } E_1 = -2.18 \times 10^{-18}\ \text{J}$$

$$\Delta E_{4-3} = E_1\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$\Delta E_{4-3} = \left(-2.18 \times 10^{-18}\ \text{J}\right)\left(\frac{1}{3^2} - \frac{1}{4^2}\right)$$

$$= -1.06 \times 10^{-19}\ \text{J}$$

$$\Delta E_{5-3} = E_1\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$

$$\Delta E_{5-3} = \left(-2.18 \times 10^{-18}\ \text{J}\right)\left(\frac{1}{3^2} - \frac{1}{5^2}\right)$$

$$= -1.55 \times 10^{-19}\ \text{J}$$

$$\Delta E_{6-3} = E_1\left(\frac{1}{3^2} - \frac{1}{6^2}\right)$$

$$\Delta E_{6-3} = \left(-2.18\times10^{-18}\ \text{J}\right)\left(\frac{1}{3^2} - \frac{1}{6^2}\right)$$

$$= -1.82\times10^{-19}\ \text{J}$$

Paraphrase

The energies emitted for the first three Paschen transitions are 1.06×10^{-19} J, 1.55×10^{-19} J, and 1.82×10^{-19} J.

(b) ***Given***

$n = 3$

Required

wavelength (λ)

frequency (f)

Analysis and Solution

Use the energy changes calculated in (a).

From $E = hf$,

$$f = \frac{E}{h}$$

$$\lambda = \frac{c}{f}$$

$$\Delta E_{4-3} = hf_{4-3}$$

$$f_{4-3} = \frac{\Delta E_{4-3}}{h}$$

$$= \frac{1.06\times10^{-19}\ \cancel{\text{J}}}{6.63\times10^{-34}\ \cancel{\text{J}}\cdot\text{s}}$$

$$= 1.60\times10^{14}\ \text{s}^{-1}$$

$$= 1.60\times10^{14}\ \text{Hz}$$

$$\lambda_{4-3} = \frac{c}{f_{4-3}}$$

$$= \frac{3.00\times10^{8}\ \frac{\text{m}}{\cancel{\text{s}}}}{1.60\times10^{14}\ \cancel{\text{s}^{-1}}}$$

$$= 1.88\times10^{-6}\ \text{m}$$

$$\Delta E_{5-3} = hf_{5-3}$$

$$f_{5-3} = \frac{\Delta E_{5-3}}{h}$$

$$= \frac{1.55\times10^{-19}\ \cancel{\text{J}}}{6.63\times10^{-34}\ \cancel{\text{J}}\cdot\text{s}}$$

$$= 2.34\times10^{14}\ \text{s}^{-1}$$

$$= 2.34\times10^{14}\ \text{Hz}$$

$$\lambda_{5-3} = \frac{c}{f_{5-3}}$$
$$= \frac{3.00 \times 10^{8}\ \frac{\text{m}}{\not{\text{s}}}}{2.34 \times 10^{14}\ \not{\text{s}}^{-1}}$$
$$= 1.28 \times 10^{-6}\ \text{m}$$
$$\Delta E_{6-3} = hf_{6-3}$$
$$f_{6-3} = \frac{\Delta E_{6-3}}{h}$$
$$= \frac{1.82 \times 10^{-19}\ \not{\text{J}}}{6.63 \times 10^{-34}\ \not{\text{J}} \cdot \text{s}}$$
$$= 2.74 \times 10^{14}\ \text{s}^{-1}$$
$$= 2.74 \times 10^{14}\ \text{Hz}$$
$$\lambda_{6-3} = \frac{c}{f_{6-3}}$$
$$= \frac{3.00 \times 10^{8}\ \frac{\text{m}}{\not{\text{s}}}}{2.74 \times 10^{14}\ \not{\text{s}}^{-1}}$$
$$= 1.09 \times 10^{-6}\ \text{m}$$

Paraphrase

The wavelengths of the first three Paschen transitions are 1.88×10^{-6} m, 1.28×10^{-6} m, and 1.09×10^{-6} m. The frequencies of the first three Paschen transitions are 1.60×10^{14} Hz, 2.34×10^{14} Hz, and 2.74×10^{14} Hz.

(c) ***Given***

$n_{\text{initial}} = 5$

$n_{\text{final}} = 3$

Required

photon energy (ΔE)

Analysis and Solution

Use the equation $E = hf$ and the value for frequency for the 5–3 transition, calculated in part (b).

$$E = hf$$
$$= \left(6.63 \times 10^{-34}\ \text{J} \cdot \not{\text{s}}\right)\left(2.34 \times 10^{14}\ \not{\text{s}}^{-1}\right)$$
$$= 1.55 \times 10^{-19}\ \text{J}$$

Paraphrase

The energy of the photon produced is 1.55×10^{-19} J. This answer is consistent with the answer in part (a) because energy varies directly as frequency.

(d) The Paschen lines are part of the infrared spectrum. They are formed by transitions that either originate from or end at the $n = 3$ energy level.

23. (a) Transitions A and C involve collisions with other atoms or electrons. Transition B is the result of photon emission because it represents a small release of energy.

(b) The difference in energy in transition B is:

$$\begin{aligned}\Delta E &= 20.66 \text{ eV} - 18.70 \text{ eV} \\ &= 1.96 \text{ eV}\end{aligned}$$

$$\Delta E = hf = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{\Delta E}$$

$$\lambda = \frac{\left(6.63 \times 10^{-34} \ \cancel{\text{J}} \cdot \cancel{\text{s}}\right)\left(3.00 \times 10^{8} \ \frac{\text{m}}{\cancel{\text{s}}}\right)}{1.96 \ \cancel{\text{eV}} \times \frac{1.60 \times 10^{-19} \ \cancel{\text{J}}}{1 \ \cancel{\text{eV}}}}$$

$$= 6.34 \times 10^{-7} \text{ m}$$

$$= 634 \text{ nm}$$

24. ***Given***

v = 10 km/s = 1.0×10^4 m/s

$\vec{B}$ = 0.25 T [out of page]

Required

the electric field $\left(\vec{E}\right)$

Analysis and Solution

Use the equation $v = \frac{\left|\vec{E}\right|}{\left|\vec{B}\right|}$.

$$\begin{aligned}\left|\vec{E}\right| &= v\left|\vec{B}\right| \\ &= (1.0 \times 10^4 \text{ m/s})(0.25 \text{ T}) \\ &= 2.5 \times 10^3 \text{ N/C}\end{aligned}$$

By the right-hand rule, the magnetic force points down (toward the bottom of the page), so the electric field must point up (toward the top of the page).

Paraphrase

An electric field of 2.5×10^3 N/C [up] will allow the alpha particle to pass undeflected through the magnetic field.

Extensions

25. (a) ***Given***

$$P = -\frac{2kq^2a^2}{3c^3}$$

Required

kinetic energy of the electron in the ground state (E_k)

Analysis and Solution

Use the law of conservation of energy to determine the kinetic energy of the electron.

For the energy of the electron in the ground state, $E_n = -\frac{E_1}{n^2}$, where

$E_1 = 2.18 \times 10^{-18}$ J and n = 1.

$$E_1 = -\frac{2.18 \times 10^{-18}\ \text{J}}{1^2}$$
$$= -2.18 \times 10^{-18}\ \text{J}$$

The potential energy of the electron is $E_\text{p} = \frac{kq_1q_2}{d}$, where $q_1 = -1.60 \times 10^{-19}\ \text{C}$, $q_2 = +1.60 \times 10^{-19}\ \text{C}$ (a proton in the hydrogen atom's nucleus and an electron have opposite charges), and $d = r_1 = 5.29 \times 10^{-11}\ \text{m}$ (the Bohr radius).

$$E_\text{p} = \frac{\left(8.99 \times 10^9\ \frac{\text{N} \cdot \text{m}^{\not{2}}}{\not{\text{C}}^{2}}\right)\left(-1.60 \times 10^{-19}\ \not{\text{C}}\right)\left(+1.60 \times 10^{-19}\ \not{\text{C}}\right)}{5.29 \times 10^{-11}\ \not{\text{m}}}$$
$$= -4.35 \times 10^{-18}\ \text{J}$$

Use $E_\text{k} + E_\text{p} = E_1$ to find E_k.

$$E_\text{k} = E_1 - E_\text{p}$$
$$= -2.18 \times 10^{-18}\ \text{J} - \left(-4.35 \times 10^{-18}\ \text{J}\right)$$
$$= 2.17 \times 10^{-18}\ \text{J}$$

Paraphrase

The kinetic energy of the electron in the ground state is $2.17 \times 10^{-18}\ \text{J}$.

(b) ***Given***

$$\frac{v^2}{r} = a$$

Required

the acceleration of an electron in the ground state (a)

Analysis and Solution

Calculate the electron's speed using the equation $E_\text{k} = \frac{1}{2}mv^2$, where $m_\text{e} = 9.11 \times 10^{-31}\ \text{kg}$, and the value for kinetic energy you calculated in part (a).

$$v = \sqrt{\frac{2E_\text{k}}{m}}$$
$$= \sqrt{\frac{2\left(2.17 \times 10^{-18}\ \text{J}\right)}{9.11 \times 10^{-31}\ \text{kg}}}$$
$$= 2.18 \times 10^6\ \text{m/s}$$

Then use the equation $\frac{v^2}{r} = a$. Substitute the value for speed you calculated above. Recall that, in the Bohr model, the hydrogen atom in its ground state has a radius of $5.29 \times 10^{-11}\ \text{m}$.

$$a = \frac{v^2}{r}$$
$$= \frac{\left(2.18 \times 10^6\ \text{m/s}\right)^2}{5.29 \times 10^{-11}\ \text{m}}$$
$$= 9.01 \times 10^{22}\ \text{m/s}^2$$

Paraphrase

The acceleration of the electron in the hydrogen atom's ground state is $9.01\times10^{22}\ \text{m/s}^2$.

(c) ***Given***

$$P=-\frac{2kq^2a^2}{3c^3}$$

Required

Show that P has the units J/s.

Analysis and Solution

Do a dimensional analysis in P.

$$-\frac{2kq^2a^2}{3c^3}=\frac{\left[\frac{\text{N}\cdot\text{m}^2}{\cancel{\text{C}^2}}\right]\left[\cancel{\text{C}^2}\right]\left[\frac{\text{m}}{\text{s}^2}\right]^2}{\left[\frac{\text{m}}{\text{s}}\right]^3}$$

$$=\left[\frac{\frac{\text{N}\cdot\text{m}^{\cancel{4}}}{\text{s}^{\cancel{4}}}}{\frac{\cancel{\text{m}^3}}{\cancel{\text{s}^3}}}\right]$$

$$=\left[\frac{\text{N}\cdot\text{m}}{\text{s}}\right]$$

$$=\left[\frac{\text{J}}{\text{s}}\right]$$

Paraphrase

Joules per second are the units of energy divided by time.

(d) ***Given***

$$P=-\frac{2kq^2a^2}{3c^3}$$

Required

time required for the electron to radiate all of its kinetic energy (Δt)

Analysis and Solution

Substitute into the equation $P=-\frac{2kq^2a^2}{3c^3}$. Then solve for time.

$$P=\frac{-2kq^2a^2}{3c^3}$$

$$=\frac{-2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\ \text{C}\right)^2\left(9.01\times10^{22}\,\text{m/s}^2\right)^2}{3\left(3.00\times10^{8}\ \text{m/s}\right)^3}$$

$$=-4.60\times10^{-8}\ \text{J/s}$$

The negative sign signifies a loss of energy.

Recall from section 6.4 that power is the rate of doing work:

$$P = \frac{E_k}{\Delta t}$$

$$\Delta t = \frac{E_k}{P}$$

Substitute this value for power and the value for kinetic energy from (a) to solve for time.

$$\Delta t = \frac{2.17 \times 10^{-18}\ \cancel{\text{J}}}{4.60 \times 10^{-8}\ \frac{\cancel{\text{J}}}{\text{s}}}$$

$$= 4.72 \times 10^{-11}\ \text{s}$$

Paraphrase

It will take the electron 4.72×10^{-11} s to give off all of its kinetic energy.

(e) Classical models of the hydrogen atom predict that an electron in the ground state should radiate energy at a rate of 4.60 × 10^{-8} J/s, which implies that the "orbiting" electron would lose its kinetic energy in 4.72 × 10^{-11} s! Stable atoms would not exist! Because matter is mostly stable, classical models of the atom are invalid.

