

Pearson Physics Level 30
Unit VIII Atomic Physics: Chapter 16
Solutions

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Example 16.1 Practice Problems

1. **Given**

$$Z = 12$$

$$A = 24$$

Required

neutron number (N)

Analysis and Solution

$$\text{Since } A = Z + N,$$

$$N = A - Z$$

$$= 24 - 12$$

$$= 12$$

Paraphrase

There are 12 neutrons in a nucleus of ${}_{12}^{24}\text{Mg}$.

2. **Given**

$$Z = 92$$

$$N = 146$$

Required

atomic mass number (A)

Analysis and Solution

$$\text{Since } A = Z + N,$$

$$A = 92 + 146$$

$$= 238$$

Paraphrase

The atomic mass number of the uranium atom is 238.

Concept Check

All three nuclei have the same atomic number, $Z = 6$, but different atomic masses and numbers of neutrons.

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Example 16.2 Practice Problems

1. **Given**

$$r = 5 \text{ fm} = 5 \times 10^{-15} \text{ m}$$

Required

gravitational force (F_g)

Analysis and Solution

Use the equation $F_g = \frac{Gm_1m_2}{r^2}$, where m_1 and m_2 represent the mass of each proton,

1.67×10^{-27} kg .

$$F_g = \frac{\left(6.67 \times 10^{-11} \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}\right) (1.67 \times 10^{-27} \text{ kg})^2}{(5 \times 10^{-15} \text{ m})^2}$$
$$= 7 \times 10^{-36} \text{ N}$$

Paraphrase

The gravitational force between two protons that are 5 fm apart is 7×10^{-36} N .

2. Given

$r = 5 \text{ fm} = 5 \times 10^{-15} \text{ m}$

Required

electrostatic force (F_e)

Analysis and Solution

Use the equation $F_e = \frac{kq_1q_2}{r^2}$, where q_1 and q_2 represent the charge on each proton,

1.60×10^{-19} C .

$$F_e = \frac{\left(8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) (1.60 \times 10^{-19} \text{ C})^2}{(5 \times 10^{-15} \text{ m})^2}$$
$$= 9 \text{ N}$$

Paraphrase

The electrostatic force that two protons exert on each other when they are 5 fm apart is 9 N.

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Example 16.3 Practice Problems

1. Given

$m = 0.221 \text{ u}$

Required

energy equivalent in eV

Analysis and Solution

Simply multiply 0.221 u by the appropriate equivalence factor:

$$0.221 \cancel{\text{u}} \times \frac{931.5 \text{ MeV}}{1 \cancel{\text{u}}} = 206 \text{ MeV}$$

Paraphrase

A mass of 0.221 u is equivalent to 206 MeV.

2. Given

$E = 250 \text{ MeV}$

Required

mass equivalent (u)

Analysis and Solution

Use the conversion factor $1 \text{ u} = 931.5 \text{ MeV}$

$$250 \text{ MeV} \times \frac{1 \text{ u}}{931.5 \text{ MeV}} = 0.268 \text{ u}$$

Paraphrase

250 MeV is equivalent to a mass of 0.268 u.

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Concept Check

Start with the expression $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$.

Note that a hydrogen atom consists of one proton and one electron. So,

$$Zm_{\text{H}} = Z(\text{mass of proton} + \text{mass of electron}).$$

Also, $m_{\text{atom}} = (\text{mass of nucleus} + \text{mass of } Z \text{ electrons})$, because there are Z electrons in a neutral atom.

Putting these equations together gives:

$\Delta m = Z(\text{mass of proton} + \text{mass of electron}) + Nm_{\text{neutron}} - (\text{mass of nucleus} + \text{mass of } Z \text{ electrons})$. The electron masses subtract, leaving $\Delta m = Zm_{\text{proton}} + Nm_{\text{neutron}} - m_{\text{nucleus}}$.

Example 16.4 Practice Problems

1. Given

$$m_{\text{Na}} = 22.989\,769 \text{ u}$$

Required

mass defect (Δm)

Analysis and Solution

From ${}_{11}^{23}\text{Na}$, $A = 23$ and $Z = 11$.

Use the equation $N = A - Z$ to find the number of neutrons, then use the equation

$$\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}, \text{ where } m_{\text{H}} = 1.007\,825 \text{ u and } m_{\text{neutron}} = 1.008\,665 \text{ u}.$$

Solution

$$N = A - Z$$

$$= 23 - 11$$

$$= 12$$

$$\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$$

$$= 11(1.007\,825 \text{ u}) + 12(1.008\,665 \text{ u}) - 22.989\,769 \text{ u}$$

$$= 0.200\,286 \text{ u}$$

Paraphrase

The mass defect for the sodium nucleus is 0.200 286 u.

2. Given

the sodium atom, ${}_{11}^{23}\text{Na}$

Required

binding energy (E_b)

Analysis and Solution

Use Δm from question 1 and the conversion factors:

$$1 \text{ u} = 1.492 \times 10^{-10} \text{ J}$$

$$1 \text{ u} = 931.5 \text{ MeV}$$

$$E_b = 0.200286 \cancel{\mu} \times \frac{1.492 \times 10^{-10} \text{ J}}{1 \cancel{\mu}} \quad \text{or} \quad = 0.200286 \cancel{\mu} \times \frac{931.5 \text{ MeV}}{1 \cancel{\mu}}$$
$$= 2.988 \times 10^{-11} \text{ J} \qquad \qquad \qquad = 186.6 \text{ MeV}$$

Paraphrase

The binding energy of the sodium-23 nucleus is $2.988 \times 10^{-11} \text{ J}$ or 186.6 MeV.

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16.1 Check and Reflect

Knowledge

1. To find the number of neutrons, N , and the number of protons, Z , use the equation

$$A = Z + N.$$

(a) ${}_{38}^{90}\text{Sr}$

$$A = 90 \text{ and } Z = 38, \text{ so}$$

$$N = A - Z$$

$$= 90 - 38$$

$$= 52$$

${}_{38}^{90}\text{Sr}$ has 38 protons and 52 neutrons.

(b) ${}_{6}^{13}\text{C}$

$$A = 13 \text{ and } Z = 6, \text{ so}$$

$$N = A - Z$$

$$= 13 - 6$$

$$= 7$$

${}_{6}^{13}\text{C}$ has 6 protons and 7 neutrons.

(c) ${}_{26}^{56}\text{Fe}$

$$A = 56 \text{ and } Z = 26, \text{ so}$$

$$N = A - Z$$

$$= 56 - 26$$

$$= 30$$

${}_{26}^{56}\text{Fe}$ has 26 protons and 30 neutrons.

(d) ${}_{1}^1\text{H}$

$$A = 1 \text{ and } Z = 1, \text{ so}$$

$$N = A - Z$$

$$= 1 - 1$$

$$= 0$$

${}_{1}^1\text{H}$ has 1 proton and no neutrons.

$$2. \frac{1.6 \times 10^{-10} \cancel{\mu}}{1.60 \times 10^{-19} \frac{\cancel{\mu}}{\text{eV}}} = 1.0 \times 10^9 \text{ eV} = 1.0 \text{ GeV}$$

$$3. 1 \text{ u} = 931.5 \text{ MeV so}$$

$$\begin{aligned} 0.25 \text{ u} &= 0.25 \cancel{\mu} \times \frac{931.5 \text{ MeV}}{1 \cancel{\mu}} \\ &= 233 \text{ MeV} \\ &= 2.3 \times 10^8 \text{ eV} \end{aligned}$$

4. **Given**

$$E = 5.00 \text{ GJ} = 5.00 \times 10^9 \text{ J}$$

Required

mass (m)

Analysis and Solution

Use the equation $E = mc^2$.

$$\begin{aligned} m &= \frac{E}{c^2} \\ &= \frac{5.00 \times 10^9 \text{ J}}{(3.00 \times 10^8 \text{ m/s})^2} \\ &= 5.56 \times 10^{-8} \text{ kg} \end{aligned}$$

Paraphrase

The quantity of mass that is converted to 5.00 GJ of energy is $5.56 \times 10^{-8} \text{ kg}$.

5. Isotopes are atoms that have the same atomic number but different neutron numbers. They are chemically very similar but do not have the same atomic mass.

6. A stable nucleus is bound together by the strong nuclear force. Binding energy is the energy required to separate all the protons and neutrons in a nucleus and move them infinitely far apart. The mass equivalence of this binding energy, calculated using Einstein's equation, $E = mc^2$, accounts for the slightly lower mass of a stable nucleus than that given by $Zm_{\text{proton}} + Nm_{\text{neutron}}$.

Applications

7. **Given**

$${}_{10}^{22}\text{Ne}, m_{\text{atom}} = 21.991\,385 \text{ u}$$

Required

binding energy (E_b)

Analysis and Solution

First calculate the number of neutrons using $A = N + Z$.

Then use the equation $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$ to calculate the mass difference.

Use your answer for the mass difference and $1 \text{ u} = 931.5 \text{ MeV}$ to calculate the binding energy.

Solution

$$\text{From } {}_{10}^{22}\text{Ne}, Z = 10, A = 22$$

$$\begin{aligned} N &= A - Z \\ &= 22 - 10 \\ &= 12 \end{aligned}$$

$$\begin{aligned}\Delta m &= 10(1.007\,825\text{ u}) + 12(1.008\,665\text{ u}) - 21.991\,385\text{ u} \\ &= 0.190\,845\text{ u} \\ 0.190\,845\text{ u} &\times \frac{931.5\text{ MeV}}{1\text{ u}} = 177.8\text{ MeV}\end{aligned}$$

Paraphrase

The binding energy for neon-22 is 177.8 MeV.

8. (a) Given

$${}_{19}^{40}\text{K}, m_{\text{atom}} = 39.963\,998\text{ u}$$

Required

mass defect (Δm)

Analysis and Solution

Use the equation $A = N + Z$ to calculate the number of neutrons.

From ${}_{19}^{40}\text{K}$, $A = 40$ and $Z = 19$, so

$$\begin{aligned}N &= A - Z \\ &= 40 - 19 \\ &= 21\end{aligned}$$

Use $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$ to find the mass defect.

$$\begin{aligned}\Delta m &= 19(1.007\,825\text{ u}) + 21(1.008\,665\text{ u}) - 39.963\,998\text{ u} \\ &= 0.366\,642\text{ u}\end{aligned}$$

Paraphrase

Potassium-40 has a mass defect of 0.366 642 u.

(b) Given

$$\Delta m = 0.366\,642\text{ u (from part (a))}$$

Required

binding energy per nucleon $\left(\frac{E_{\text{b}}}{A}\right)$

Analysis and Solution

Convert the mass defect to binding energy using:

$$\begin{aligned}E_{\text{b}} &= 0.366\,642\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\ &= 341.53\text{ MeV}\end{aligned}$$

Divide by atomic mass number to obtain the binding energy per nucleon, $\frac{E_{\text{b}}}{A}$.

From ${}_{19}^{40}\text{K}$, $A = 40$, so

$$\begin{aligned}\frac{E_{\text{b}}}{A} &= \frac{341.53\text{ MeV}}{40} \\ &= 8.538\text{ MeV/nucleon}\end{aligned}$$

Paraphrase

Potassium-40 has a binding energy per nucleon of 8.538 MeV.

9. To estimate the binding energy, read values from the graph. Note that the binding energy is given in energy per nucleon, so multiply by the atomic mass number to obtain the binding energy per nucleus.

(a) From the graph in Figure 16.4, ${}_{6}^{13}\text{C}$ has ~ 7.7 MeV/nucleon, so

$$E_b = 13 \text{ nucleons} \times 7.7 \text{ MeV/nucleon} \\ = 100 \text{ MeV}$$

(b) ${}^{56}_{26}\text{Fe}$ has 8.5 MeV/nucleon, so

$$E_b = 56 \text{ nucleons} \times 8.5 \text{ MeV/nucleon} \\ = 476 \text{ MeV}$$

(c) ${}^{238}_{92}\text{U}$ has 7.3 MeV/nucleon, so

$$E_b = 238 \text{ nucleons} \times 7.3 \text{ MeV/nucleon} \\ = 1737 \text{ MeV}$$

10. MeV is an energy unit, so it is equivalent to joules. The units of MeV/c^2 are

$$\frac{\text{J}}{\left(\frac{\text{m}}{\text{s}}\right)^2} = \frac{\text{N} \cdot \cancel{\text{m}}}{\frac{\text{m}^{\cancel{2}}}{\text{s}^2}} \\ = \frac{\text{N} \cdot \text{s}^2}{\text{m}} \\ = \frac{\text{kg} \cdot \frac{\cancel{\text{m}}}{\cancel{\text{s}^2}} \cdot \cancel{\text{s}^2}}{\cancel{\text{m}}} \\ = \text{kg}$$

Extensions

11. (a) The strong nuclear force is the stronger of the two forces, by a factor of about 100 but, unlike the electrostatic force, it acts only over very short distances—a few femtometres at most—and acts on both protons and neutrons. The range of the electromagnetic force is infinite, but it acts only on charged particles, such as protons and electrons.
- (b) The protons repel each other by the electromagnetic force and are attracted to each other and neutrons by the strong nuclear force. If the strong force acted over a larger distance, then much larger nuclei would exist. If the electromagnetic force were stronger, then it would be difficult to form large nuclei, especially those with many protons.
12. The nucleus is in a constant struggle between the strong nuclear force that binds it together and the electrostatic force that causes like charges to repel. If the electrostatic force were stronger, then nuclei would be less stable, and very large nuclei would probably not exist because the strong nuclear force would not be able to overcome the repulsion of the protons in a large nucleus.

13. **Given**

$$r = r_0 A^{\frac{1}{3}}, \text{ where } r_0 = 1.20 \text{ fm}$$

Required

radius of the nucleus of ${}^{90}_{38}\text{Sr}$ atom (r)

distance between adjacent nucleons in ${}^{90}_{38}\text{Sr}$ nucleus (d)

Analysis and Solution

From ${}^{90}_{38}\text{Sr}$, $A = 90$. Substitute this value into the equation for nuclear radius.

The radius of the nucleus of the strontium-90 atom is:

$$r = (1.20 \text{ fm})90^{\frac{1}{3}} \approx 5.38 \text{ fm}$$

The distance d is twice the radius of the space occupied by one nucleon, V_{nucleon} . To estimate V_{nucleon} , first determine the volume of the nucleus and divide it by 90, the number of nucleons in the nucleus. This gives the volume of the space occupied by one nucleon, V_{nucleon} .

The volume of the nucleus is:

$$\begin{aligned} V &= \frac{4}{3}\pi r^3 \\ &= \frac{4}{3}\pi[(1.20 \text{ fm})90^{\frac{1}{3}}]^3 \\ &= \frac{4}{3}\pi(1.20 \text{ fm})^3 90 \\ V_{\text{nucleon}} &= \frac{V}{A} \\ &= \frac{\frac{4}{3}\pi(1.20 \text{ fm})^3 90}{90} \\ &= \frac{4}{3}\pi(1.20 \text{ fm})^3 \end{aligned}$$

But,

$$\begin{aligned} V_{\text{nucleon}} &= \frac{4}{3}\pi r_{\text{nucleon}}^3 \\ \frac{4}{3}\pi r_{\text{nucleon}}^3 &= \frac{4}{3}\pi(1.20 \text{ fm})^3 \\ r_{\text{nucleon}} &= 1.20 \text{ fm} \end{aligned}$$

$$\begin{aligned} d &= 2r_{\text{nucleon}} \\ &= 2 \times 1.20 \text{ fm} \\ &\sim 2 \text{ fm} \end{aligned}$$

Paraphrase

The radius of the ${}^{90}_{38}\text{Sr}$ atom is about 5.38 fm. The centres of the nucleons are, on average, about 2 fm apart. This number sets an upper limit on the possible size of the individual nucleons. The actual size of the protons and neutrons must be less than this value.

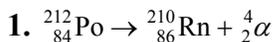
Concept Check

Apply the right-hand rule for a positive charge. The thumb points in the direction of charge motion, to the right. The fingers point out of the page, in the direction of the magnetic field. The palm faces down, toward the bottom of the page, and represents the direction of the magnetic force. Since the alpha particle moves toward the bottom of the

page in Figure 16.5, in the same direction as the magnetic force, it must have a positive charge. The beta particle is deflected in the opposite direction, toward the top of the page. It must therefore have a negative charge, according to the left-hand rule. The alpha particle is deflected much less than the beta particle, so it must be considerably more massive than the beta particle. The gamma ray appears to be neutral because it is not deflected.

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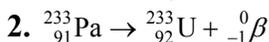
Example 16.5 Practice Problems



Charge: $84 \neq 86 + 2$

Atomic mass number: $212 \neq 210 + 4$

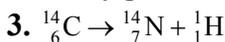
This equation violates the laws of conservation of charge and of atomic mass number. Therefore, this decay process is impossible.



Charge: $91 = 92 - 1$

Atomic mass number: $233 = 233$

This equation is correct—both mass number and charge are conserved. Therefore, this decay process is possible.



Charge: $6 \neq 7 + 1$

Atomic mass number: $14 \neq 14 + 1$

This equation violates the laws of conservation of charge and of atomic mass number. Therefore, this decay process is impossible.

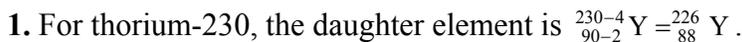
Concept Check

The conservation law applies to nucleons, which make up most of the atom's mass. Electrons are not nucleons.

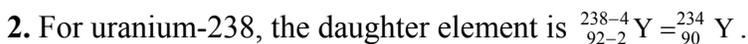
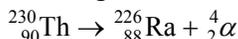
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Example 16.6 Practice Problems

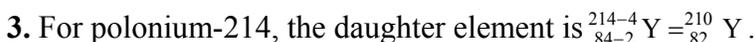
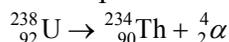
All alpha decay processes fit the pattern ${}_Z^AX \rightarrow {}_{Z-2}^{A-4}Y + {}_2^4\alpha$.



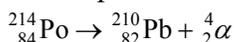
On the periodic table, the element with $Z = 88$ is radium.



On the periodic table, the element with $Z = 90$ is thorium.



On the periodic table, the element with $Z = 82$ is lead.



Concept Check

Alpha decay will only occur if the nucleus can achieve a lower energy by emitting an alpha particle. The alpha particle removes energy from the nucleus, so ΔE must be positive. Use the mass-energy equivalence to understand this concept. The mass-energy of the parent nucleus equals the mass-energy of the daughter nucleus and the alpha particle, plus the kinetic energy of these two particles, according to the equation

$$m_{\text{parent}}c^2 = m_{\text{daughter}}c^2 + m_{\alpha}c^2 + \Delta E$$

where ΔE is the kinetic energy of the daughter nucleus and of the alpha particle.

Example 16.7 Practice Problems

1. Given



Required

energy released during α -decay (ΔE)

Analysis and Solution

From ${}_Z^AX \rightarrow {}_{Z-2}^{A-4}Y + {}_2^4\alpha$, the α -decay process for thorium is ${}_{90}^{230}\text{Th} \rightarrow {}_{88}^{226}\text{Ra} + {}_2^4\alpha$

(see Example 16.6 Practice Problem 1).

The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the products.

$$\begin{aligned} \Delta m &= m_{\text{parent}} - m_{\text{products}} \\ &= m_{{}_{90}^{230}\text{Th}} - (m_{{}_{88}^{226}\text{Ra}} + m_{{}_2^4\alpha}) \\ &= 230.033\,134\text{ u} - 226.025\,410\text{ u} - 4.002\,603\text{ u} \\ &= 0.005\,121\text{ u} \\ \Delta E &= 0.005\,121\text{ u} \times \frac{1.492 \times 10^{-10}\text{ J}}{1\text{ u}} \quad \text{or} \quad 0.005\,121\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\ &= 7.641 \times 10^{-13}\text{ J} \qquad \qquad \qquad = 4.770\text{ MeV} \end{aligned}$$

Paraphrase

A thorium nucleus releases $7.641 \times 10^{-13}\text{ J}$ or 4.770 MeV of energy when it undergoes α -decay.

2. Given



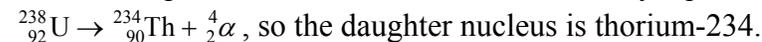
Required

energy released during α -decay (ΔE)

Analysis and Solution

Determine the decay process of uranium-238 to find the daughter nucleus.

From Example 16.6 Practice Problem 2, the decay equation for uranium-238 is



The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the products.

$$\begin{aligned}
\Delta m &= m_{\text{parent}} - m_{\text{products}} \\
&= m_{\text{}_{92}^{238}\text{U}} - (m_{\text{}_{90}^{234}\text{Th}} + m_{\text{}_{2}^4\alpha}) \\
&= 238.050\,788\text{ u} - 234.043\,601\text{ u} - 4.002\,603\text{ u} \\
&= 0.004\,584\text{ u} \\
\Delta E &= 0.004\,584\text{ u} \times \frac{1.492 \times 10^{-10}\text{ J}}{1\text{ u}} \quad \text{or} \quad = 0.004\,584\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\
&= 6.839 \times 10^{-13}\text{ J} \qquad \qquad \qquad = 4.270\text{ MeV}
\end{aligned}$$

Paraphrase

Uranium releases $6.839 \times 10^{-13}\text{ J}$ or 4.270 MeV of energy when it undergoes α -decay.

3. Given



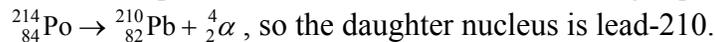
Required

energy released during α -decay (ΔE)

Analysis and Solution

Determine the decay process of polonium-214 to find the daughter nucleus.

From Example 16.6 Practice Problem 3, the decay equation for polonium-214 is



The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the products.

$$\begin{aligned}
\Delta m &= m_{\text{parent}} - m_{\text{products}} \\
&= m_{\text{}_{84}^{214}\text{Po}} - (m_{\text{}_{82}^{210}\text{Pb}} + m_{\text{}_{2}^4\alpha}) \\
&= 213.995\,201\text{ u} - 209.984\,189\text{ u} - 4.002\,603\text{ u} \\
&= 0.008\,409\text{ u} \\
\Delta E &= 0.008\,409\text{ u} \times \frac{1.492 \times 10^{-10}\text{ J}}{1\text{ u}} \quad \text{or} \quad = 0.008\,409\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\
&= 1.255 \times 10^{-12}\text{ J} \qquad \qquad \qquad = 7.833\text{ MeV}
\end{aligned}$$

Paraphrase

When polonium undergoes α -decay, it releases $1.255 \times 10^{-12}\text{ J}$ or 7.833 MeV of energy.

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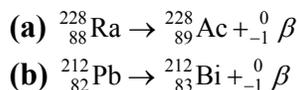
Concept Check

Free neutrons are unstable and decay into a proton, electron, and anti-neutrino. In addition to creating these three particles, about $1.2 \times 10^{-13}\text{ J}$ of energy is released (as kinetic energy). By using mass-energy equivalence, this tiny bit of energy explains why the neutron has a slightly higher rest mass. The typical lifetime of a free neutron is about 900 s.

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Example 16.8 Practice Problems

1. Use the pattern ${}^A_Z\text{X} \rightarrow {}^A_{Z+1}\text{X} + {}^0_{-1}\beta$. The atomic number must increase by one without changing the atomic mass number.



Example 16.9 Practice Problems

1. (a) **Given**

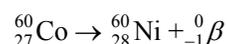
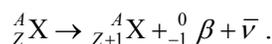
cobalt-60 nucleus

Required

products of β^- decay

Analysis and Solution

Identify the correct structure for cobalt-60 and then apply the β^- decay pattern.

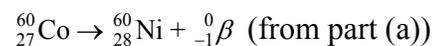


Paraphrase

The β^- decay of cobalt-60 produces ${}_{28}^{60}\text{Ni}$.

(b) **Given**

cobalt-60 nucleus



Required

energy released by β^- decay (ΔE)

Analysis and Solution

The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the decay products. Use the equation

$\Delta m = m_{\text{parent}} - m_{\text{products}}$, then convert this value to energy using Einstein's mass-energy equivalence principle.

$$\begin{aligned} \Delta m &= m_{\text{parent}} - m_{\text{products}} \\ &= m_{{}_{27}^{60}\text{Co}} - \left(m_{{}_{28}^{60}\text{Ni}} + m_{{}_{-1}^0\beta} \right) \\ &= m_{{}_{27}^{60}\text{Co}} - m_{{}_{28}^{60}\text{Ni}} \\ &= 59.933\,817\text{ u} - 59.930\,786\text{ u} \\ &= 0.003\,031\text{ u} \end{aligned}$$

$$\begin{aligned} \Delta E &= 0.003\,031\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\ &= 2.823\text{ MeV} \end{aligned}$$

Paraphrase

The β^- decay of a cobalt-60 nucleus releases 2.823 MeV of energy.

Concept Check

The discrepancy in the beta decay that led to the discovery of the neutrino was a small amount of missing energy (hence mass through the mass-energy equivalence). Because

charge is conserved in this decay, the neutrino does not change the charge. Therefore, it must be neutral.

Student Book page 805

Example 16.10 Practice Problems

1. (a) During β^+ decay, the atomic number decreases by 1. The atomic number, Z , of thallium-202 is 81. When the atomic number of thallium-202 decreases by 1, it becomes $Z - 1 = 81 - 1 = 80$, which is the atomic number for mercury-202.

(b) Use the general pattern ${}^A_Z X \rightarrow {}^A_{Z-1} Y + {}^0_1 \beta + \nu$ for β^+ decay: ${}^{202}_{81} \text{Tl} \rightarrow {}^{202}_{80} \text{Hg} + {}^0_1 \beta + \nu$

(c) **Given**

decrease in mass of the thallium nucleus = 0.001 463 u

Required

energy released in the decay (ΔE)

Analysis and Solution

The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the products.

$$\begin{aligned} \Delta m &= m_{\text{parent}} - m_{\text{products}} \\ &= m_{{}^{202}_{81}\text{Tl}} - (m_{{}^{202}_{80}\text{Hg}} + m_{{}^0_1\beta}) \\ &= m_{{}^{202}_{81}\text{Tl}} - (m_{{}^{202}_{80}\text{Hg}} + m_{{}^0_{-1}\beta} + m_{{}^0_1\beta}) \\ &= m_{{}^{202}_{81}\text{Tl}} - (m_{{}^{202}_{80}\text{Hg}} + 2m_{{}^0_{-1}\beta}) \end{aligned}$$

You are given the decrease in mass of the thallium-202 nucleus, so the equation can be written as

$$\begin{aligned} \Delta m &= (m_{{}^{202}_{81}\text{Tl}} - m_{{}^{202}_{80}\text{Hg}}) - 2m_{{}^0_{-1}\beta} \\ &= 0.001\,463\text{ u} - 2(0.000\,549\text{ u}) \\ &= 0.000\,365\text{ u} \end{aligned}$$

Now use the equation for mass-energy equivalence.

$$\begin{aligned} \Delta E &= 0.000\,365 \cancel{\mu} \times \frac{931.5\text{ MeV}}{1 \cancel{\mu}} \\ &= 0.3400\text{ MeV} \end{aligned}$$

Paraphrase

The energy released by the β^+ decay of thallium-202 is 0.3400 MeV.

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Concept Check

Gamma decays cannot be shown as paths on a decay series graph because gamma decay does not change either the atomic number (Z) or the atomic mass number (A) of an atom. Gamma decay is the result of nucleons moving to lower-energy states and emitting gamma-ray photons.

16.2 Check and Reflect

Knowledge

- The three basic radioactive decay processes are:
 - alpha decay, where the nucleus emits an alpha particle and its atomic number changes from Z to $Z - 2$
 - beta decay:
 - β^- decay, where the nucleus emits an electron and an antineutrino, and its atomic number increases by 1
 - β^+ decay, where the nucleus emits a positron and a neutrino, and its atomic number decreases by 1
 - gamma decay, where the nucleus drops from an excited state to a lower energy state and emits a gamma-ray photon
- The heaviest stable isotopes tend to have a slightly higher number of neutrons than protons, so the ratio is roughly 3:2 (see Figure 16.10 on p. 806 of the student text).
- (a) Use the pattern ${}^A_Z X \rightarrow {}^{A-4}_{Z-2} Y + {}^4_2 \alpha$ to get ${}^{234}_{91} \text{Pa} \rightarrow {}^{230}_{89} \text{Ac} + {}^4_2 \alpha$.
 (b) The parent nucleus is protactinium-234 and the daughter nucleus is actinium-230.
- (a) Since Z increases from 6 to 7, this type of beta decay must be β^- decay.
 (b) ${}^{14}_6 \text{C} \rightarrow {}^{14}_7 \text{N} + {}^0_{-1} \beta + \bar{\nu}$
- (a) Since Z decreases by 1, the decay must be β^+ decay.
 (b) ${}^{22}_{11} \text{Na} \rightarrow {}^{22}_{10} \text{Ne} + {}^0_1 \beta + \nu$
- The emission of the beta particle leaves the daughter nucleus in an excited state. The gamma-ray photon is emitted as the daughter nucleus drops into a lower energy state.
- Of the three main radioactive decay processes, gamma rays usually have the greatest penetrating power.

Applications

8. Given

${}^{22}_{11} \text{Na}$ decays to produce ${}^{22}_{10} \text{Ne}$.

$$m_{\text{Na}} = 21.994\,436 \text{ u}$$

$$m_{\text{Ne}} = 21.991\,385 \text{ u}$$

Required

energy released by the decay process (ΔE)

Analysis and Solution

The atomic number decreases by 1, so ${}^{22}_{11} \text{Na}$ undergoes β^+ decay to produce ${}^{22}_{10} \text{Ne}$

according to the equation ${}^{22}_{11} \text{Na} \rightarrow {}^{22}_{10} \text{Ne} + {}^0_1 \beta + \nu$.

Find the mass defect, Δm , using the equation $\Delta m = m_{\text{parent}} - m_{\text{products}}$, and convert it to its energy equivalent by using the conversion factor $1 \text{ u} = 931.5 \text{ MeV}$.

$$\begin{aligned}
\Delta m &= m_{\text{parent}} - m_{\text{products}} \\
&= m_{11}^{22}\text{Na} - \left(m_{10}^{22}\text{Ne} + m_{1}^{0}\beta \right) \\
&= m_{11}^{22}\text{Na} - \left(m_{10}^{22}\text{Ne} + m_{-1}^{0}\beta + m_{1}^{0}\beta \right) \\
&= m_{11}^{22}\text{Na} - \left(m_{10}^{22}\text{Ne} + 2m_{-1}^{0}\beta \right) \\
&= 21.994\,436\text{ u} - 21.991\,385\text{ u} - 2(0.000\,549\text{ u}) \\
&= 0.001\,953\text{ u}
\end{aligned}$$

$$\begin{aligned}
\Delta E &= 0.001\,953\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\
&= 1.819\text{ MeV}
\end{aligned}$$

Paraphrase

The decay of sodium into neon liberates 1.819 MeV of energy.

- Yes, atomic number can increase during nuclear decay. A β^- decay causes atomic number to increase by one. For example, carbon-14 can decay to nitrogen-14, according to the decay process ${}^{14}_6\text{C} \rightarrow {}^{14}_7\text{N} + {}^0_{-1}\beta + \bar{\nu}$.
- You receive about 300 μSv of radiation from natural sources each year and about 75 μSv from medical and dental sources. Hence, you receive on average about four times more radiation from natural background sources than from medical procedures.
-

Decay process	Decay type	Parent element	Daughter element
(a) ${}^{232}_{90}\text{Th} \rightarrow {}^{228}_{88}\text{Ra}^* + {}^4_2\alpha$	alpha decay	thorium	radium
(b) ${}^{22}_{11}\text{Na} \rightarrow {}^{22}_{10}\text{Ne} + {}^0_1\beta + \nu$	β^+ decay	sodium	neon
(c) ${}^{228}_{88}\text{Ra}^* \rightarrow {}^{228}_{88}\text{Ra} + \gamma$	gamma decay	radium	radium
(d) ${}^{228}_{88}\text{Ra} \rightarrow {}^{228}_{89}\text{Ac} + {}^0_{-1}\beta + \bar{\nu}$	β^- decay	radium	actinium
(e) ${}^{228}_{89}\text{Ac} \rightarrow {}^{228}_{90}\text{Th} + {}^0_{-1}\beta + \bar{\nu}$	β^- decay	actinium	thorium
(f) ${}^{228}_{90}\text{Th} \rightarrow {}^{224}_{88}\text{Ra} + {}^4_2\alpha$	alpha decay	thorium	radium
(g) ${}^1_1\text{p} \rightarrow {}^1_0\text{n} + {}^0_1\beta + \nu$	β^+ decay	proton	neutron

Extensions

- (a) The absorption of an electron causes a decrease in the atomic number. The electron combines with a proton in the nucleus to form a neutron.
 (b) ${}^A_Z\text{X} + {}^0_{-1}\beta \rightarrow {}^A_{Z-1}\text{Y} + \nu$
 (c) Electron capture is similar to beta decay because both processes change an atom from one element to another. However, during electron capture, an electron is absorbed, whereas during beta decay, an electron (or a positron) is emitted.
- One possible experiment would be to bombard a nucleus with neutrons in an attempt to excite the nucleus and cause it to emit gamma-ray photons when the nucleus returns to the ground state. If the nucleus behaves in a manner analogous to the atom, you should see distinct energies given off in the form of gamma-ray photons of specific wavelengths, similar to the bright line spectrum seen in electron transitions.
- Radon is a natural radioactive decay product of radium. Since radium occurs naturally, so does radon. Concrete contains trace amounts of radium and hence can

also produce trace amounts of radon. Since radon is a noble gas and cannot be held by chemical bonds, it can migrate through the cement (or soil). Furthermore, because it is heavier than air, radon can accumulate in a basement. Such accumulation is a health concern because radon is itself radioactive—it undergoes alpha decay. If you inhale radon, you run an increased risk of absorbing an alpha emitter into your lungs or bloodstream, which poses an increased risk of cancer.

Student Book page 812

Example 16.11 Practice Problems

1. **Given**

$$\lambda = 4.1 \times 10^{-9} \text{ s}^{-1}$$

$$N = 1.01 \times 10^{22} \text{ atoms}$$

Required

activity (A)

Analysis and Solution

Use the equation $A = -\lambda N$ and substitute the given values.

$$A = -(4.1 \times 10^{-9} \text{ s}^{-1})(1.01 \times 10^{22})$$

$$= -4.1 \times 10^{13} \text{ s}^{-1}$$

$$= -4.1 \times 10^{13} \text{ Bq}$$

The negative sign means that the number of cobalt-60 nuclei is decreasing.

Paraphrase

The sample has a decay rate of $4.1 \times 10^{13} \text{ Bq}$.

2. **Given**

$$N = 5.00 \times 10^{20} \text{ atoms}$$

$$A = 2.50 \times 10^{12} \text{ Bq}$$

Required

the decay constant (λ)

Analysis and Solution

Use the equation $A = -\lambda N$ (the negative sign signifies decay).

$$\lambda = -\frac{A}{N}$$

$$= -\frac{2.50 \times 10^{12} \text{ Bq}}{5.00 \times 10^{20} \text{ atoms}}$$

$$= -5.00 \times 10^{-9} \text{ s}^{-1}$$

Paraphrase

The decay constant for this sample is $5.00 \times 10^{-9} \text{ s}^{-1}$.

Student Book page 813

Example 16.12 Practice Problems

1. **Given**

$$t_{1/2} = 1.6 \text{ s}$$

Required

time required for 99% of astatine in a sample to decay (t)

Analysis and Solution

Use the equation $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$. Re-write as $\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$ and let $\alpha = \frac{t}{t_{1/2}}$:

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^\alpha$$

Since 99% of the sample has undergone decay, $\frac{N}{N_0} = 0.01$.

$$0.01 = \left(\frac{1}{2}\right)^\alpha$$

Choose values for α and plug them into your calculation until $\left(\frac{1}{2}\right)^\alpha = 0.01$.

$$\alpha = 6.64$$

Solve for t :

$$6.64 = \frac{t}{t_{1/2}}$$

$$t = (6.64) \left(t_{1/2}\right)$$

$$= (6.64)(1.6 \text{ s})$$

$$= 10.6 \text{ s}$$

Paraphrase

It takes only about 11 s for 99% of a sample of astatine-218 to undergo radioactive decay.

2. Given

$$t_{1/2} = 1600 \text{ years}$$

$$t = 8000 \text{ years}$$

Required

the percentage of a sample of radium-226 remaining after 8000 years $\left(\frac{N}{N_0}\right)$

Analysis and Solution

Use the equation $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\begin{aligned}\frac{N}{N_0} &= \left(\frac{1}{2}\right)^{\frac{8000}{1600}} \\ &= \left(\frac{1}{2}\right)^5 \\ &= \frac{1}{32} \\ &= 3.125\%\end{aligned}$$

Paraphrase

After 8000 years, only 3.125% of the original radium-226 sample will remain.

Student Book page 814

Example 16.13 Practice Problems

1. Given

$$t_{1/2} = 29.1 \text{ years}$$

$$t = 100 \text{ years}$$

Required

proportion of the sample remaining after 100 years $\left(\frac{N}{N_0}\right)$

Analysis and Solution

Use the equation $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$.

$$\begin{aligned}\frac{N}{N_0} &= \left(\frac{1}{2}\right)^{t/t_{1/2}} \\ &= \left(\frac{1}{2}\right)^{\frac{100}{29.1}} \\ &= \left(\frac{1}{2}\right)^{3.44} \\ &= 0.0924 \\ &= 9.24\%\end{aligned}$$

Paraphrase

After 100 years, 9.24% of the original amount of strontium-90 remains.

2. Given

$$t_{1/2} = 12.3 \text{ years}$$

$$m = 100 \text{ mg}$$

$$t = 5.0 \text{ years}$$

Required

proportion of tritium remaining after 5.0 years $\left(\frac{N}{N_0}\right)$

Analysis and Solution

Use the equation $N = N_0 \left(\frac{1}{2}\right)^{t/T_{1/2}}$.

$$\begin{aligned}\frac{N}{N_0} &= \left(\frac{1}{2}\right)^{t/T_{1/2}} \\ &= \left(\frac{1}{2}\right)^{\frac{5.0}{12.3}} \\ &= \left(\frac{1}{2}\right)^{0.41} \\ &= 0.75 \\ &= 75\%\end{aligned}$$

75% of 100 mg is 75 mg.

Paraphrase

After 5.0 years, 75%, or 75 mg, of tritium will remain.

Student Book page 816

Concept Check

Most materials weakly absorb beta radiation. The amount of absorption depends on the thickness of the absorber. Therefore, by measuring the amount of radiation emitted by a known source of beta radiation, the thickness of the absorber can be determined. Gamma radiation can easily penetrate most materials, and it takes very large thickness differences to produce an appreciable difference in the absorption of gamma rays. Because of its great penetrating quality, gamma radiation can be used to provide a detailed image of the interior of such objects as metal pipes and panels, to help reveal flaws or cracks in structures.

Student Book page 817

16.3 Check and Reflect

Knowledge

1. For each half-life, one-half of a sample undergoes decay while the other half remains.

So, after four half-lives, there remains $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$ of the original amount of radioactive material.

2. **Given**

$$\begin{aligned}\lambda &= 5.8 \times 10^{-12} \text{ s}^{-1} \\ N &= 6.4 \times 10^{23} \text{ atoms}\end{aligned}$$

Required

activity of the sample (A)

Analysis and Solution

Substitute the given values into the equation $A = -\lambda N$.

$$\begin{aligned}
 A &= -\lambda N \\
 &= -(5.8 \times 10^{-12} \text{ s}^{-1})(6.4 \times 10^{23}) \\
 &= -3.7 \times 10^{12} \text{ s}^{-1}
 \end{aligned}$$

Paraphrase

The radioactive sample decays at a rate of 3.7×10^{12} atoms/s.

3. Since both samples have the same mass, assume that they have roughly the same number of atoms. The first sample has a half-life that is much shorter than that of the second sample. Therefore, the first sample must undergo more radioactive decays per second, and hence have a greater activity, than the second sample.

Applications

4. **Given**

$$\frac{N}{N_0} = \frac{1}{16}$$

$$t_{1/2} = 3.0 \times 10^5 \text{ years}$$

Required

the age of the rock sample (t)

Analysis and Solution

Use the equation $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$$

$$\frac{1}{16} = \left(\frac{1}{2}\right)^4$$

$$\frac{t}{t_{1/2}} = 4$$

$$t = 4 \left(t_{1/2}\right)$$

$$= 4(3.0 \times 10^5 \text{ years})$$

$$= 1.2 \times 10^6 \text{ years}$$

Paraphrase

The rock sample is approximately 1.2 million years old.

5. **Given**

$$t_{1/2} = 2.6 \text{ h}$$

$$t = 24 \text{ h}$$

Required

the proportion of the tracer remaining in a patient after 24 h $\left(\frac{N}{N_0}\right)$

Analysis and Solution

Use the equation $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$.

$$\begin{aligned}\frac{N}{N_0} &= \left(\frac{1}{2}\right)^{t/t_{1/2}} \\ &= \left(\frac{1}{2}\right)^{24/2.6} \\ &= \left(\frac{1}{2}\right)^{9.23} \\ &= 1.7 \times 10^{-3}\end{aligned}$$

Paraphrase

After 24 h, only 1.7×10^{-3} (0.17%) of the original quantity of the tracer remains.

6. Given

$$\frac{N}{N_0} = 25\% = \frac{1}{4}$$

Required

the age of the arrow (t)

Analysis and Solution

Assume that the arrow has one-quarter of its original carbon-14 content.

Use the equation $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$.

Look up the half-life of carbon-14 in a source. It is 5730 years.

$$\begin{aligned}\frac{N}{N_0} &= \left(\frac{1}{2}\right)^{t/t_{1/2}} \\ \frac{1}{4} &= \left(\frac{1}{2}\right)^2 \\ \frac{t}{t_{1/2}} &= 2 \\ t &= 2 \left(t_{1/2}\right) \\ &= 2(5730 \text{ years}) \\ &= 11\,460 \text{ years} \\ &= 1.1 \times 10^4 \text{ years}\end{aligned}$$

The arrow appears to be two half-lives old.

Paraphrase

The arrow is approximately 1.1×10^4 years old.

7. Given

$$A = 2.5 \text{ MBq}$$

$$t_{1/2} = 12 \text{ h} = 0.5 \text{ d}$$

Required

the activity of the sample one week (7 d) later (A)

Analysis and Solution

The activity depends on the number of radioactive nuclei present ($A = -\lambda N$), but N depends on time, according to the equation $N = N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$. Therefore, $A = -\lambda N_0 \left(\frac{1}{2}\right)^{t/t_{1/2}}$.

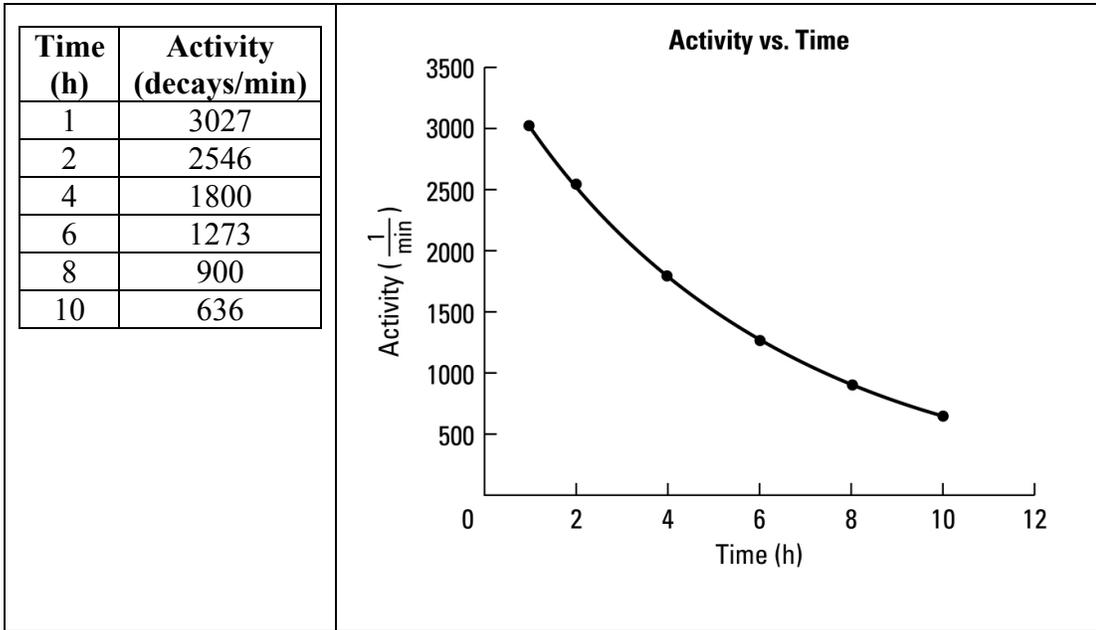
Let the original activity be $A_0 = -\lambda N_0$.

$$\begin{aligned} A &= A_0 \left(\frac{1}{2}\right)^{t/t_{1/2}} \\ &= (2.5 \times 10^6 \text{ Bq}) \left(\frac{1}{2}\right)^{7/0.5} \\ &= 153 \text{ Bq} \\ &= 1.5 \times 10^2 \text{ Bq} \end{aligned}$$

Paraphrase

After one week, the activity will have dropped from 2.5×10^6 Bq to just 1.5×10^2 Bq.

8.



(a) The activity drops from 3000 decays/min to 1500 decays/min in about 4 h, so the half-life is about 4 h.

(b) **Given**

$$A = 3027 \text{ at } t = 1 \text{ h}$$

$$t_{1/2} = 4 \text{ h}$$

Required

activity at $t = 0$ (A_0)

Analysis and Solution

From question 7, $A = A_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\begin{aligned} A_0 &= \frac{A}{\left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}} \\ &= \frac{3027}{\left(\frac{1}{2}\right)^{\frac{1}{4}}} \\ &= 3600 \end{aligned}$$

Paraphrase

At $t = 0$, the activity of the sample is 3600 decays/min.

Extensions

9. Carbon-14 is useless for dating extremely old artefacts because it has a half-life of only 5730 years. A 65-million-year-old object would be about 11 284 carbon-14 half-lives old! If you started with one mole of carbon-14, then after 11 284 half-lives, there would be no carbon-14 atoms left! You should be suspicious because carbon-14 could not have been used to date this bone.

10.

For	Against
Irradiation kills bacteria, thereby prolonging the shelf-life of foods and allowing them to be transported longer distances without spoiling.	Irradiation may cause changes to the genetic structure of food. The major argument against irradiation is that its long-term effects are unknown.

11. (a) Depleted uranium is uranium left over from spent fuel rods or from decommissioned nuclear weapons.
- (b) Uranium is a very heavy, dense metal. Its high density makes it very effective at piercing armour. It is also useful as ballast because it takes up less room while still providing much needed weight (due to its high density).
- (c) There are at least three major concerns with the use of depleted uranium.
- Depleted uranium is a heavy metal and therefore presents the same kinds of health concerns that other, including non-radioactive, heavy metals pose for health, including nervous system disorders.
 - Depleted uranium is still radioactive. Exposure to it poses the same health risks as exposure to any radioactive substance does.
 - Depleted uranium can be physically changed by impact into a dust, especially when used in weaponry. The dust, when airborne and inhaled, poses an even greater health risk.

Concept Check

If the binding energy per nucleon increases, then the nucleons are more tightly bound to each other. In order for nucleons to bind more tightly, they must each lose energy.

Consequently, a nuclear reaction that increases the binding energy per nucleon releases energy. As an analogy, imagine that you dropped your backpack while hiking and it rolled down the side of an embankment, ending up 100 m below you. Your backpack is now more tightly bound to Earth because it is closer to it, and it will take even more energy to bring it back to your elevation.

Binding energy represents the energy needed to separate nucleons from each other. The nucleus with the highest binding energy per nucleon is iron, which makes it the most stable of nuclei.

Student Book page 819

Example 16.14 Practice Problems

1. Given

Initial mass: ${}_{92}^{235}\text{U}$ plus one neutron

Final mass: ${}_{40}^{94}\text{Zr}$, ${}_{52}^{139}\text{Te}$, and three neutrons

Required

energy released (ΔE)

Analysis and Solution

Use the atomic mass data on p. 881 of the student text to calculate the net change in mass resulting from the reaction.

$$\begin{aligned}
 m_i &= m_{{}_{92}^{235}\text{U}} + m_n \\
 &= 235.043\,930\text{ u} + 1.008\,665\text{ u} \\
 &= 236.052\,595\text{ u} \\
 m_f &= m_{{}_{40}^{94}\text{Zr}} + m_{{}_{52}^{139}\text{Te}} + 3\,{}^1_0\text{n} \\
 &= 93.906\,315\text{ u} + 138.934\,700\text{ u} + 3(1.008\,665\text{ u}) \\
 &= 235.867\,010\text{ u} \\
 m_i - m_f &= 236.052\,595\text{ u} - 235.867\,010\text{ u} \\
 &= 0.185\,585\text{ u}
 \end{aligned}$$

Use mass-energy equivalence to calculate the energy released.

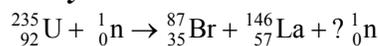
1 u is equivalent to 931.5 MeV/u, so

$$\begin{aligned}
 \Delta E &= 0.185\,585\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\
 &= 172.9\text{ MeV}
 \end{aligned}$$

Paraphrase

The nuclear fission of U-235 into tellurium and zirconium releases 172.9 MeV of energy.

2. Analysis and Solution

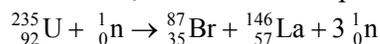


To determine the number of neutrons released, balance the atomic mass numbers on both sides of the equation:

$$235 + 1 = 87 + 146 + ?$$

$$? = 3$$

Therefore, the balanced equation for the reaction is



3. Given

Initial mass: ${}_{92}^{235}\text{U}$ plus one neutron

Final mass: ${}_{35}^{87}\text{Br}$, ${}_{57}^{146}\text{La}$, and three neutrons

Required

energy released (ΔE)

Analysis and Solution

Use the atomic mass data on p. 881 of the student text to calculate the net change in mass resulting from the reaction.

$$\begin{aligned}m_i &= {}_{92}^{235}\text{U} + m_n \\ &= 235.043\,930\text{ u} + 1.008\,665\text{ u} \\ &= 236.052\,595\text{ u} \\ m_f &= {}_{35}^{87}\text{Br} + {}_{57}^{146}\text{La} + 3\,{}^1_0\text{n} \\ &= 86.920\,711\text{ u} + 145.925\,791\text{ u} + 3(1.008\,665\text{ u}) \\ &= 235.872\,497\text{ u} \\ m_i - m_f &= 236.052\,595\text{ u} - 235.872\,497\text{ u} \\ &= 0.180\,098\text{ u}\end{aligned}$$

Use mass-energy equivalence to calculate the energy released.

1 u is equivalent to 931.5 MeV/u, so

$$\begin{aligned}\Delta E &= 0.180\,098\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\ &= 167.8\text{ MeV}\end{aligned}$$

Paraphrase

This reaction releases 167.8 MeV of energy.

Student Book page 820

Example 16.15 Practice Problems

1. (a) Given

$$E = 1600\text{ MJ}$$

$$d = 500\text{ km}$$

Required

the mass of gasoline (m)

Analysis and Solution

The chemical energy in gasoline is $4.4 \times 10^7\text{ J/kg}$ (from Example 16.15).

Divide the given energy (1600 MJ) by this value.

$$\frac{1600 \times 10^6\text{ J}}{4.4 \times 10^7\text{ J/kg}} = 36\text{ kg}$$

Paraphrase

It takes 36 kg of gasoline to provide 1600 MJ of energy.

(b) Given

$$E = 1600\text{ MJ}$$

$$d = 500\text{ km}$$

Required

mass of U-235 required to deliver the same amount of energy (m)

Analysis and Solution

Do the same calculation as in part (a) using the energy content for U-235:

7.10×10^{13} J/kg (from Example 16.15).

$$\frac{1600 \times 10^6 \cancel{\text{J}}}{7.10 \times 10^{13} \frac{\cancel{\text{J}}}{\text{kg}}} = 2.3 \times 10^{-5} \text{ kg} = 23 \text{ mg}$$

Paraphrase

To travel 500 km in an average family car would require only 23 mg of uranium-235!

Student Book page 821
Concept Check

The small nuclear dimensions mean that the de Broglie wavelengths of nucleons are much smaller than the de Broglie wavelengths of electrons in the atom. Since momentum depends inversely on the wavelength, nucleons have much higher momenta, and hence higher energy, than do the electrons in the atom. The particle-in-a-box example from Chapter 14 (Example 14.12) could be used here as an analogy. Nucleons are confined to much smaller “boxes”, and hence must have greater energies.

Another reason why nuclear reactions are more energetic has to do with the strong nuclear force, which operates only in atomic nuclei. It takes a great deal of energy to overcome the strong force. Also, the electric potential energy of protons in a nucleus is enormous because the protons are packed so tightly together. Changing nuclear structure involves manipulating these forces, which can lead to enormous energy changes.

Student Book page 822
Example 16.16 Practice Problems**1. (a) Given**

$$P = 1.6 \times 10^{25} \text{ W}$$

Required

number of helium nuclei produced per second

Analysis and Solution

Use the results of Example 16.16. Every fusion of four hydrogen atoms to a helium nucleus releases 26.71 MeV. Convert this value to joules and then divide power (J/s) by energy to determine the number of helium nuclei formed each second.

$$26.71 \times 10^6 \text{ eV} \times 1.60 \times 10^{-19} \text{ J/eV} = 4.274 \times 10^{-12} \text{ J}$$

$$\begin{aligned} \text{Rate} &= \frac{1.6 \times 10^{25} \text{ J/s}}{4.274 \times 10^{-12} \text{ J}} \\ &= 3.7 \times 10^{36} \text{ s}^{-1} \end{aligned}$$

Paraphrase

The star produces 3.7×10^{36} new helium nuclei each second by nuclear fusion.

(b) Given $t = 4$ billion years**Required**mass of helium produced (m)**Analysis and Solution**

Multiply the result in (a) by the time (4 billion years) to get the total number of the nuclei formed.

$$t = 4 \times 10^9 \frac{\text{years}}{\text{year}} \times \frac{365 \text{ days}}{\text{year}} \times \frac{24 \text{ h}}{\text{day}} \times \frac{3600 \text{ s}}{\text{h}} = 1.26 \times 10^{17} \text{ s}$$

$$\begin{aligned} \text{The number of helium nuclei produced} &= (3.7 \times 10^{36} \text{ s}^{-1})(1.26 \times 10^{17} \text{ s}) \\ &= 4.7 \times 10^{53} \end{aligned}$$

Multiply by the mass of a helium nucleus, 6.65×10^{-27} kg, to obtain the total mass of helium produced.

$$\begin{aligned} \text{The mass of helium produced} &= (4.7 \times 10^{53} \text{ nuclei})(6.65 \times 10^{-27} \text{ kg/nucleus}) \\ &= 3.1 \times 10^{27} \text{ kg} \end{aligned}$$

Paraphrase

After 4 billion years, this star will have produced 3.1×10^{27} kg of helium.

Student Book page 823**Concept Check**

For fusion to occur, protons must be close enough so that the strong nuclear force can cause them to fuse. In order to come sufficiently close, the protons must first overcome the repulsive electrostatic force between them, called the Coulomb barrier. To penetrate the Coulomb barrier, they must have very high kinetic energy, that is, they must be moving very fast, which is the same as stating that they must be at a very high temperature. For this reason, fusion reactions can occur only at extremely high temperatures.

Student Book page 824**Concept Check**

Tritium is a radioactive isotope of hydrogen. If inhaled, it poses a serious health concern because tritium undergoes beta decay. If absorbed into the body, tritium becomes a long-term, low-level radiation source that can cause cellular damage, including genetic mutation. Because tritium is a form of hydrogen, it can form chemical bonds with other elements that can be incorporated into organic molecules in our tissues.

16.4 Check and Reflect**Knowledge**

1. (a) The total number of nucleons must add up to 235. There are 140 nucleons in the Xe nucleus, plus two more neutrons, so

$$\begin{aligned} A &= 235 - 140 - 2 \\ &= 93 \end{aligned}$$

Also, charge must be conserved, so

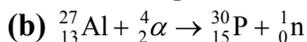
$$92 = 54 + Z$$

$$Z = 38$$

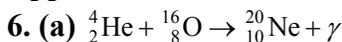
From the periodic table, the missing element is strontium-93, ${}_{38}^{93}\text{Sr}$.

- (b) This reaction is a spontaneous fission reaction because a larger nucleus splits into two smaller fragments.
2. In order for energy to be released during a nuclear reaction, the binding energy per nucleon must increase. The release of energy means that the nucleons in the products of a nuclear reaction are more tightly bound per nucleon than the nucleons in the parent nucleus.
3. The combined binding energy of the iron and silicon nuclei is
 $492 \text{ MeV} + 237 \text{ MeV} = 729 \text{ MeV}$
 Because this value is greater than the binding energy of the final nucleus, 718 MeV, this reaction requires energy, so energy will not be released.
4. (a) Heavy elements ($A > 120$) will release energy in fission reactions because they can split into nuclei that each have binding energies per nucleon that are higher than the binding energies of the parent nucleus.
 (b) Light elements ($A < 50$) that can combine to form a nucleus with a higher binding energy per nucleon than the original nuclei are most likely to undergo fusion.
5. (a) Use the alpha decay pattern, ${}^A_Z\text{X} \rightarrow {}^{A-4}_{Z-2}\text{Y} + {}^4_2\alpha$. Aluminium-27 is ${}^{27}_{13}\text{Al}$, so the absorption equation is ${}^{27}_{13}\text{Al} + {}^4_2\alpha \rightarrow {}^A_Z\text{Y} + {}^1_0\text{n}$.
 $27 + 4 = A + 1$
 $A = 30$
 $13 + 2 = Z + 0$
 $Z = 15$

From the periodic table, this element is phosphorus.



Applications



This fusion reaction produces neon-20. Since the neon-20 nucleus will be in an excited state, a gamma ray will be emitted.

(b) **Given**

helium-4

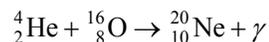
oxygen-16

Required

energy released (ΔE)

Analysis and Solution

Use the nuclear process equation and the atomic mass data on page 881 of the student text to calculate the net change in mass resulting from the reaction.



$$\begin{aligned} m_i &= m_{\text{He}} + m_{\text{O}} \\ &= 4.002\,603 \text{ u} + 15.994\,915 \text{ u} \end{aligned}$$

$$\begin{aligned} m_f &= m_{\text{Ne}} \\ &= 19.992\,440 \text{ u} \end{aligned}$$

$$m_i - m_f = 4.002\,603\text{ u} + 15.994\,915\text{ u} - 19.992\,440\text{ u}$$

$$= 0.005\,078\text{ u}$$

Use mass-energy equivalence to calculate the energy released.

1 u is equivalent to 931.5 eV, so

$$\Delta E = 0.005\,078 \cancel{\text{ u}} \times \frac{931.5\text{ MeV}}{\cancel{\text{ u}}}$$

$$= 4.730\text{ MeV}$$

Paraphrase

The fusion reaction of helium-4 and oxygen-16 releases 4.730 MeV of energy.

7. (a) Given

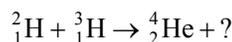
Deuterium (${}^2_1\text{H}$) and tritium (${}^3_1\text{H}$) fuse to form helium (${}^4_2\text{He}$).

Required

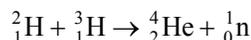
particle emitted

Analysis and Solution

Use the laws of conservation of atomic mass number and of charge.



Since there is no change in charge but a change in atomic mass number, a neutron has been emitted. The fusion equation is therefore



Paraphrase

This reaction emits a neutron.

(b) Given

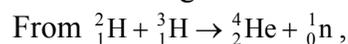
Deuterium (${}^2_1\text{H}$) and tritium (${}^3_1\text{H}$) fuse to form helium (${}^4_2\text{He}$).

Required

energy released (ΔE)

Analysis and Solution

Use the atomic mass data on p. 881 of the student text to calculate the net change in mass resulting from the reaction.



$$m_i = {}^2_1\text{H} + {}^3_1\text{H}$$

$$= 2.014\,102\text{ u} + 3.016\,049\text{ u}$$

$$m_f = {}^4_2\text{He} + {}^1_0\text{n}$$

$$= 4.002\,603\text{ u} + 1.008\,665\text{ u}$$

$$m_i - m_f = 2.014\,102\text{ u} + 3.016\,049\text{ u} - (4.002\,603\text{ u} + 1.008\,665\text{ u})$$

$$= 0.018\,883\text{ u}$$

Use mass-energy equivalence to calculate the energy released.

1 u is equivalent to 931.5 MeV, so

$$\Delta E = 0.018\,883 \cancel{\text{ u}} \times \frac{931.5\text{ MeV}}{\cancel{\text{ u}}}$$

$$= 17.59\text{ MeV}$$

Paraphrase

The reaction produces 17.59 MeV of energy.

8. (a) Given

$$P = 700 \text{ MW}$$

$$\text{efficiency} = 27\%$$

$$\text{average energy release per fission} = 200 \text{ MeV}$$

Required

number of fission reactions per second

Analysis and Solution

First determine how much nuclear power must be generated to produce 700 MW of electrical power.

$$P \times 0.27 = 700 \times 10^6 \text{ W}$$

$$P = 2.6 \times 10^9 \text{ W}$$

$$= 2.6 \times 10^9 \text{ J/s}$$

Divide the energy needed each second by 200 MeV per fission to get the number of U-235 nuclei needed each second.

$$\frac{2.6 \times 10^9 \frac{\text{J}}{\text{s}}}{(200 \times 10^6 \text{ eV}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right)} = 8.1 \times 10^{19} \text{ s}^{-1}$$

Paraphrase

The number of nuclei that undergo fission each second when the reactor is running at full power is 8.1×10^{19} .

(b) Given

A nuclear reactor produces 700 MW of electrical power. The average energy release per fission is 200 MeV and CANDU is 27% efficient.

Required

mass of U-235 used per year (m)

Analysis and Solution

Multiply the answer in (a) by the total time (one year) and by the mass per U-235 nucleus. In one year, the number of fission reactions that occur is

$$\left(8.1 \times 10^{19} \frac{1}{\text{s}} \right) \left(365.25 \frac{\text{d}}{\text{a}} \right) \left(86\,400 \frac{\text{s}}{\text{d}} \right) = 2.556 \times 10^{27} \text{ a}^{-1}$$

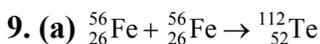
The mass of U-235 used is:

$$m = (2.556 \times 10^{27}) (235.043\,930 \text{ u}) \left(1.660\,539 \times 10^{-27} \frac{\text{kg}}{\text{u}} \right)$$

$$= 1.0 \times 10^3 \text{ kg}$$

Paraphrase

The CANDU reactor will consume one tonne of U-235 in a year to produce an electrical power output of 700 MW. The assumption made is that only pure U-235 is used. In fact, the U-235 content is much smaller, so a considerably greater mass of nuclear fuel must be used.

Extensions

The element formed is tellurium-112.

(b) Given

$$m_{\text{112Te}} = 111.917\ 010\ \text{u}$$

Required

Show that this reaction absorbs energy.

Analysis and Solution

Use the atomic mass data on p. 881 of the student text to calculate the net change in mass resulting from the reaction.

$$\begin{aligned} m_i &= m_{\text{26Fe}}^{56} + m_{\text{26Fe}}^{56} \\ &= 55.934\ 938\ \text{u} + 55.934\ 938\ \text{u} \\ &= 111.869\ 876\ \text{u} \end{aligned}$$

$$\begin{aligned} m_f &= m_{\text{52Te}}^{112} \\ &= 111.917\ 010\ \text{u} \end{aligned}$$

$$\begin{aligned} m_i - m_f &= 111.869\ 876\ \text{u} - 111.917\ 010\ \text{u} \\ &= -0.047\ 134\ \text{u} \end{aligned}$$

Use mass-energy equivalence to calculate the energy released.

1 u is equivalent to 931.5 MeV, so

$$\begin{aligned} \Delta E &= -0.047\ 134\ \cancel{\text{u}} \times \frac{931.5\ \text{MeV}}{\cancel{\text{u}}} \\ &= -43.91\ \text{MeV} \end{aligned}$$

A negative answer means that energy is absorbed.

Paraphrase

The fusion of two iron-56 nuclei results in the formation of tellurium-112, which is heavier than the initial two nuclei. Therefore, this reaction absorbs 43.91 MeV of energy.

- (c) Stars similar to our Sun do not have sufficient mass to reach the core temperatures and densities needed to cause the production of iron, let alone the fusion of iron. The production of elements that are heavier than iron occurs during supernova explosions. The Sun is not massive enough to undergo a supernova explosion.

10. (a) The major radioactive waste isotopes produced by nuclear reactors, and their half-lives, are listed in the table below.

Waste Isotope	Half-life
cesium-137	30 years
strontium-90	30 years
uranium-235	700 million years
plutonium-240	24 000 years

- (b) A current short-term method of storing nuclear waste is the use of on-site, sealed containers. Highly radioactive materials are stored in water-filled storage tanks. A long-term storage method is vitrification, where radioactive material is stored in a glass-like matrix as “logs” that can be deposited in deep wells bored in a stable landmass such as the Canadian Shield.

One proposed long-term disposal method, called subductive waste disposal, would place radioactive waste in subduction zones along active undersea faults. The radioactive material would be subducted or absorbed in the magma layers of Earth’s mantle.

11.

Coal power (risk/benefits)	Nuclear power (risk/benefits)
Benefit: Coal provides a relatively low-cost energy source that is available in many areas of the world and is easy to mine. Production of power from coal is an old and proven technology.	Risk: Nuclear power is a complex and costly technology to implement. Incidents such as Three Mile Island in the U.S. and Chernobyl in the Ukraine remind us that this method of power generation is potentially dangerous. Nuclear accidents can pollute large regions of a country with wastes that have half-lives of tens to thousands of years.
Risk: Burning coal is a major source of greenhouse gases, which pose an environmental risk.	Benefit: Nuclear power is a “clean” energy source with few direct pollutants introduced into the environment (not including waste disposal, uranium mining, or a potential nuclear accident).
Risk: Coal combustion releases pollutants into the atmosphere, including naturally occurring radioactive isotopes. Without costly “scrubbing” of exhaust gas from a coal generation plant, more radioactive material is introduced into the environment than direct emissions from a nuclear power plant.	Risk: Disposal of spent fuel rods and radioactive waste is a major environmental concern for which no fool-proof long-term method currently exists.
Benefit: Coal is a safe energy source in politically unstable regions of the world.	Risk: Nuclear power generation poses a number of potential risks in politically unstable regions of the world. Acts of terrorism could lead to sabotage of nuclear facilities. As well, nuclear reactors can be modified to become “breeder reactors” for producing weapons-grade fissionable material.
Risk: Coal is a non-renewable resource that requires large-scale pit or sub-surface mining. Both methods present environmental hazards.	Benefit: Although uranium is also a non-renewable resource, it is many times more efficient as an energy source than coal combustion.

Student Book pages 826–827

Chapter 16 Review

Knowledge

1. Not all nuclei contain more neutrons than protons. For example, a hydrogen-1 nucleus contains no neutrons, hydrogen-2 contains one proton and one neutron, and helium-4 contains two protons and two neutrons.

2. In most cases, the atomic mass number for an atom is greater than the atomic number.
The exception is hydrogen-1, where $A = Z$.

3. Elements with the same atomic number but different neutron numbers are isotopes.

4. These nuclei are isotopes of uranium. Each atom contains the same number of electrons and protons but different numbers of neutrons.

5. For ${}^{115}_{55}\text{Cs}$, $A = Z + N$.

$$N = 115 - 55 = 60 \text{ neutrons}$$

$$Z = 55 \text{ protons}$$

6. $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$ so

$$\begin{aligned} 50 \text{ MeV} &= \left(50 \times 10^6 \cancel{\text{ eV}}\right) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{ eV}}}\right) \\ &= 8.0 \times 10^{-12} \text{ J} \end{aligned}$$

7. Use the equation $E = mc^2$.

$$E = (0.001 \text{ kg})(3.00 \times 10^8 \text{ m/s})^2$$

$$= 9 \times 10^{13} \text{ J}$$

One gram of matter is equivalent to $9 \times 10^{13} \text{ J}$ of energy.

8. $1 \text{ u} = 1.660\,539 \times 10^{-27} \text{ kg}$

The energy equivalent of this mass is

$$E = mc^2$$

$$= (1.660\,539 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2$$

$$= 1.49 \times 10^{-10} \text{ J}$$

$$2.3 \text{ u} = (2.3)(1.49 \times 10^{-10} \text{ J})$$

$$= 3.4 \times 10^{-10} \text{ J}$$

The energy equivalent of 2.3 u is $3.4 \times 10^{-10} \text{ J}$.

9. Use the conversion factor $1 \text{ u} = 931.5 \text{ MeV}$.

$$300 \cancel{\text{ MeV}} \times \frac{1 \text{ u}}{931.5 \cancel{\text{ MeV}}} = 0.322 \text{ u}$$

10. Use binding energy as the energy equivalent of the mass defect.

Since $1 \text{ u} = 931.5 \text{ MeV}$,

$$0.022 \cancel{\text{ u}} \times \frac{931.5 \text{ MeV}}{\cancel{\text{ u}}} = 20 \text{ MeV}$$

11. Gamma decay processes do not change the atomic number of a nucleus.

12. (a) Beta particles can be negative (electrons) or positive (positrons).

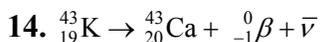
(b) An alpha particle is a helium nucleus, so its charge is +2.

(c) A gamma ray is a high-energy photon, so it has no charge.

13. (a) Alpha decay emits a helium nucleus, which removes two neutrons and two protons from the nucleus.

(b) β^- decay appears as the emission of an electron and the appearance of a new proton in the nucleus. β^+ decay appears as the emission of a positron and the creation of a new neutron in the nucleus.

(c) Gamma decay does not change the number of nucleons. It represents the change in internal energy of the nucleus.



This equation is an example of β^- decay due to the emission of an electron and an antineutrino. The parent nucleus is potassium and the daughter element is calcium.

15. Gamma radiation is the most penetrating. Alpha radiation is the least penetrating.

16. The most important evidence involves the energy of radioactive decay products. Typical radioactive decays are measured in hundreds of thousands to millions of electron volts. Atomic processes are many orders of magnitude less energetic. Also, one element changes into another element during radioactive decay. This is evidence that radioactivity originates in the nucleus, where the protons are located. If ionization and redox reactions involved nucleons, chemical reactions would create new elements.

17. Radon exposure averages approximately $200 \mu\text{Sv/year}$, compared to $73 \mu\text{Sv/year}$ for dental X rays. Of these two sources, radon exposure is much more significant because radon is a gas that, when inhaled, can be absorbed into body tissue. Radon decays via alpha particle emission and remains in the body for long periods of time, in close proximity to vital organs.

18. **Given**

$$N = 1.5 \times 10^{20}$$

$$\lambda = 1.2 \times 10^{-12} \text{ s}^{-1}$$

Analysis and Solution

Use the definition for activity,

$$\begin{aligned} A &= \frac{\Delta N}{\Delta t} = -\lambda N \\ &= -(1.2 \times 10^{-12} \text{ s}^{-1})(1.5 \times 10^{20}) \\ &= -1.8 \times 10^8 \text{ s}^{-1} \\ &= 1.8 \times 10^8 \text{ decays/s} \end{aligned}$$

19. **Given**

$$N_0 = 5.0 \times 10^{20}$$

$$t_{1/2} = 1.5 \text{ h}$$

$$t = 6 \text{ h}$$

Required

the number of nuclei remaining after 6 h (N)

Analysis and Solution

The number of radioactive nuclei has decreased by exactly one-half in 1.5 h:

$$\frac{5.0 \times 10^{20}}{2.5 \times 10^{20}} = 2$$

So, the half-life, $t_{1/2}$, of the sample is 1.5 h. Since $6 \text{ h} = 4 \times 1.5 \text{ h}$, the sample will

decay by an additional 4 half-lives.

The number of radioactive nuclei remaining after 6 h is:

$$\begin{aligned} (2.5 \times 10^{20}) \left(\frac{1}{2}\right)^4 &= 1.6 \times 10^{19} \\ &= 2 \times 10^{19} \end{aligned}$$

Alternatively, use the equation $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\begin{aligned} N &= (2.5 \times 10^{20}) \left(\frac{1}{2}\right)^{\frac{6}{1.5}} \\ &= 1.6 \times 10^{19} \\ &= 2 \times 10^{19} \end{aligned}$$

Paraphrase

The sample has a half-life of 1.5 h so, after 6 h, it will have decayed from 5.0×10^{20} to 2×10^{19} nuclei.

20. The half-life of carbon-14 is 5730 years. For this reason, carbon-14 is usually useful for dating items that are no older than 50 000–60 000 years. Ages of most rock samples are measured in millions of years. Also, carbon is not commonly found in most rocks, except for carbonates.
21. Fission produces energy when a heavy nucleus splits into two smaller nuclei that have greater binding energy per nucleon than the original nucleus. This process is most effective for the most massive nuclei, with $A > 200$.
22. The fusion of hydrogen nuclei into helium is the most important energy source in stars.
23. The steps in the proton-proton chain are:
 - i) hydrogen + hydrogen produces deuterium
 - ii) hydrogen + deuterium produces helium-3
 - iii) helium-3 + helium-3 produces helium-4 plus 2 protons

Applications

24. (a) Given



Required

binding energy per nucleon (E_b)

Analysis and Solution

Use $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$ to find the mass defect, where Z = atomic number, N = neutron number, m_{H} = mass of the hydrogen atom, m_{neutron} = mass of neutron, and m_{atom} = mass of the atom.

From ${}^4_2\text{He}$, $A = 4$, $Z = 2$

$$N = A - Z$$

$$= 4 - 2$$

$$= 2$$

$$m_{{}^4_2\text{He}} = 4.002\,603\text{ u}$$

$$\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$$

$$= 2(1.007\,825\text{ u}) + 2(1.008\,665\text{ u}) - 4.002\,603\text{ u}$$

$$= 0.030\,377\text{ u}$$

Use the mass-energy equivalence to calculate binding energy from the mass defect.

$$1\text{ u} = 931.5\text{ MeV}$$

$$E_b = 0.030\,377 \cancel{\mu} \times \frac{931.5 \text{ MeV}}{\cancel{\mu}}$$

$$= 28.296 \text{ MeV}$$

Divide this number by the atomic number to find the binding energy per nucleon.

$$\frac{28.296 \text{ MeV}}{4} = 7.074 \text{ MeV}$$

Paraphrase

The binding energy per nucleon in a helium atom is 7.074 MeV.

(b) Given



Required

binding energy per nucleon (E_b)

Analysis and Solution

Use $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$ to find the mass defect, where Z = atomic number, N = neutron number, m_{H} = mass of the hydrogen atom, m_{neutron} = mass of neutron, and m_{atom} = mass of the atom. Use mass data from p. 881 of the student text.

From ${}_{14}^{28}\text{Si}$, $A = 28$, $Z = 14$

$$N = A - Z$$

$$= 28 - 14$$

$$= 14$$

$$m_{{}_{14}^{28}\text{Si}} = 27.976\,927 \text{ u}$$

$$\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$$

$$= 14(1.007\,825 \text{ u}) + 14(1.008\,665 \text{ u}) - 27.976\,927 \text{ u}$$

$$= 0.253\,933 \text{ u}$$

Use the mass-energy equivalence to calculate binding energy from the mass defect.

$$1 \text{ u} = 931.5 \text{ MeV}$$

$$E_b = 0.253\,933 \cancel{\mu} \times \frac{931.5 \text{ MeV}}{\cancel{\mu}}$$

$$= 236.539 \text{ MeV}$$

Divide this number by the atomic number to find the binding energy per nucleon.

$$\frac{236.539 \text{ MeV}}{28} = 8.448 \text{ MeV}$$

Paraphrase

The binding energy per nucleon in a silicon atom is 8.448 MeV.

(c) Given



Required

binding energy per nucleon (E_b)

Analysis and Solution

Use $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$ to find the mass defect, where Z = atomic number, N = neutron number, m_{H} = mass of the hydrogen atom, m_{neutron} = mass of neutron, and m_{atom} = mass of the atom. Use mass data from p. 881 of the student text.

text.

From ${}_{26}^{58}\text{Fe}$, $A = 58$, $Z = 26$

$$\begin{aligned}N &= A - Z \\ &= 58 - 26 \\ &= 32\end{aligned}$$

$$m_{{}_{26}^{58}\text{Fe}} = 57.933\,276\text{ u}$$

$$\begin{aligned}\Delta m &= Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}} \\ &= 26(1.007\,825\text{ u}) + 32(1.008\,665\text{ u}) - 57.933\,276\text{ u} \\ &= 0.547\,454\text{ u}\end{aligned}$$

Use the mass-energy equivalence to calculate binding energy from the mass defect.

$$1\text{ u} = 931.5\text{ MeV}$$

$$\begin{aligned}E_{\text{b}} &= 0.547\,454\cancel{\text{ u}} \times \frac{931.5\text{ MeV}}{\cancel{\text{ u}}} \\ &= 509.953\text{ MeV}\end{aligned}$$

Divide this number by the atomic number to find the binding energy per nucleon.

$$\frac{509.953\text{ MeV}}{58} = 8.792\text{ MeV}$$

Paraphrase

The binding energy per nucleon in an iron atom is 8.792 MeV.

(d) Given



Required

binding energy per nucleon (E_{b})

Analysis and Solution

Use $\Delta m = Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}}$ to find the mass defect, where Z = atomic number, N = neutron number, m_{H} = mass of the hydrogen atom, m_{neutron} = mass of neutron, and m_{atom} = mass of the atom. Use mass data from p. 881 of the student text.

From ${}_{92}^{235}\text{U}$, $A = 235$, $Z = 92$

$$\begin{aligned}N &= A - Z \\ &= 235 - 92 \\ &= 143\end{aligned}$$

$$m_{{}_{92}^{235}\text{U}} = 235.043\,930\text{ u}$$

$$\begin{aligned}\Delta m &= Zm_{\text{H}} + Nm_{\text{neutron}} - m_{\text{atom}} \\ &= 92(1.007\,825\text{ u}) + 143(1.008\,665\text{ u}) - 235.043\,930\text{ u} \\ &= 1.915\,065\text{ u}\end{aligned}$$

Use the mass-energy equivalence to calculate binding energy from the mass defect.

$$1\text{ u} = 931.5\text{ MeV}$$

$$\begin{aligned}E_{\text{b}} &= 1.915\,065\cancel{\text{ u}} \times \frac{931.5\text{ MeV}}{\cancel{\text{ u}}} \\ &= 1783.883\text{ MeV}\end{aligned}$$

Divide this number by the atomic number to find the binding energy per nucleon.

$$\frac{1783.883 \text{ MeV}}{235} = 7.591 \text{ MeV}$$

Paraphrase

The binding energy per nucleon in a uranium atom is 7.591 MeV. It is useful to note that iron has the greatest binding energy of the four nuclei.

25. (a) Given

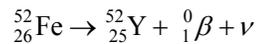
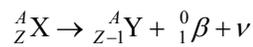


Required

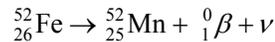
β^+ decay process

Analysis and Solution

Use the basic form for β^+ decay:



The element with $A = 52$ and $Z = 25$ is manganese. Therefore,



Paraphrase

The β^+ decay of ${}_{26}^{52}\text{Fe}$ converts iron into manganese.

(b) To demonstrate charge conservation, note that $26 = 25 + {}^0_1\beta = 25 + 1 = 26$.

The atomic mass number remains at 52. β^+ decay turns a proton into a neutron and yields a positron in the process. Charge and atomic mass number are therefore conserved.

26. (a) Given

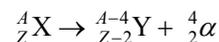
α -decay to produce lead-208

Required

the parent nucleus

Analysis and Solution

Use the standard form for α -decay.



Let $A - 4 = 208$ and $Z - 2 = 82$.

$$A = 212, Z = 84$$

Therefore, the parent nucleus is polonium-212.

Paraphrase

Polonium-212 decays into lead-208 by emitting an alpha particle.

(b) Given

α -decay to produce lead-208

Required

kinetic energy of alpha particle (E_k)

Analysis and Solution

The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the products.

$$\Delta m = m_{\text{parent}} - m_{\text{products}}$$

Use the atomic masses from p. 881 of the student text.

$$\begin{aligned}\Delta m &= m_{84}^{212}\text{Po} - (m_{82}^{208}\text{Pb} + m_2^4\text{He}) \\ &= 211.988\,868\text{ u} - 207.976\,652\text{ u} - 4.002\,603\text{ u} \\ &= 0.009\,613\text{ u} \\ 1\text{ u is equivalent to } &931.5\text{ MeV, so} \\ \Delta E &= 0.009\,613\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\ &= 8.955\text{ MeV}\end{aligned}$$

Paraphrase

The approximate kinetic energy of the alpha particle is 8.955 MeV.

27. (a) Given

phosphorus is converted into silicon

Required

decay equation

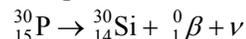
Analysis and Solution

Write both phosphorus and silicon in atomic form and inspect to determine what decay process has occurred.

phosphorus = ${}_{15}^{30}\text{P}$

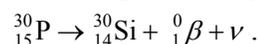
silicon = ${}_{14}^{30}\text{Si}$

Phosphorus decays into a silicon isotope. Since the atomic number decreases by 1, this process must be β^+ decay.



Paraphrase

The complete decay process for the transmutation of ${}_{15}^{30}\text{P}$ into ${}_{14}^{30}\text{Si}$ is



(b) Given

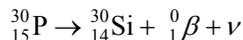
phosphorus is converted into silicon

Required

energy released (ΔE)

Analysis and Solution

The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the products.



To determine energy released, compare masses of parent and daughter nuclei.

$$\begin{aligned}\Delta m &= m_{\text{parent}} - m_{\text{products}} \\ &= m_{15}^{30}\text{P} - (m_{14}^{30}\text{Si} + m_1^0\beta) \\ &= m_{15}^{30}\text{P} - (m_{14}^{30}\text{Si} + m_{-1}^0\beta + m_1^0\beta)\end{aligned}$$

Since electrons and positrons have the same mass,

$$\begin{aligned}\Delta m &= m_{15}^{30}\text{P} - (m_{14}^{30}\text{Si} + 2m_{-1}^0\beta) \\ &= 29.978\,314\text{ u} - [29.973\,770\text{ u} + 2(0.000\,549\text{ u})] \\ &= 0.003\,446\text{ u}\end{aligned}$$

1 u is equivalent to 931.5 MeV, so

$$\begin{aligned}\Delta E &= 0.003\,446 \cancel{\mu} \times \frac{931.5 \text{ MeV}}{1 \cancel{\mu}} \\ &= 3.210 \text{ MeV}\end{aligned}$$

Paraphrase

The energy released during this decay is 3.210 MeV.

28. Given

Artifacts contain $\frac{1}{4}$ the original amount of carbon-14.

Required

approximate age of artifacts

Analysis and Solution

The half-life, $t_{1/2}$, of carbon-14 is 5730 years.

Use the equation $N = N_0 \left(\frac{1}{2}\right)^{\frac{t}{t_{1/2}}}$.

$$\frac{N}{N_0} = \frac{1}{4} = \left(\frac{1}{2}\right)^2$$

$$\frac{t}{t_{1/2}} = 2$$

$$t = 2t_{1/2}$$

$$t = 2(5730 \text{ years})$$

$$= 11\,460 \text{ years}$$

$$= 1.1 \times 10^4 \text{ years}$$

Paraphrase

The oldest archaeological sites in Alberta appear to be approximately 1.1×10^4 years old.

29. (a) Given

$$\lambda = 1.98 \times 10^{-11} \text{ s}^{-1}$$

$$A = 0.10 \text{ MBq}$$

Required

number of radium atoms (N)

Analysis and Solution

Use the equation $A = -\lambda N$.

$$\begin{aligned}N &= -\frac{A}{\lambda} \\ &= -\frac{0.10 \times 10^6 \text{ Bq}}{1.98 \times 10^{-11} \text{ s}^{-1}} \\ &= -5.1 \times 10^{15}\end{aligned}$$

(The negative sign indicates that the number of atoms is decreasing.)

Paraphrase

The clock dial contains 5.1×10^{15} atoms of radium-226.

(b) Given

5.1×10^{15} atoms of radium-226 (from part (a))

Requiredmass of radium (m)**Analysis and Solution**

The mass of a radium atom is 226 u, where $1 \text{ u} = 1.660\,539 \times 10^{-27} \text{ kg}$. Multiply the mass per atom by the number of atoms.

$$m = (226 \cancel{\text{u}}) \left(\frac{1.660\,539 \times 10^{-27} \text{ kg}}{1 \cancel{\text{u}}} \right) (5.1 \times 10^{15})$$

$$= 1.9 \times 10^{-9} \text{ kg}$$

Paraphrase

The clock dial contains $1.9 \times 10^{-9} \text{ kg}$ of radium.

(c) Given

$$t_{1/2} = 1600 \text{ years}$$

$$t = 5000 \text{ years}$$

$$\lambda = 1.98 \times 10^{-11} \text{ s}^{-1}$$

$$N_0 = 5.1 \times 10^{15} \text{ atoms of radium-226 (from part (a))}$$

Requiredactivity after 5000 years (A)**Analysis and Solution**

Use the equation $A = -\lambda N$, where $N = N_0 \left(\frac{1}{2} \right)^{t/t_{1/2}}$.

$$A = -\lambda N$$

$$= -\lambda N_0 \left(\frac{1}{2} \right)^{t/t_{1/2}}$$

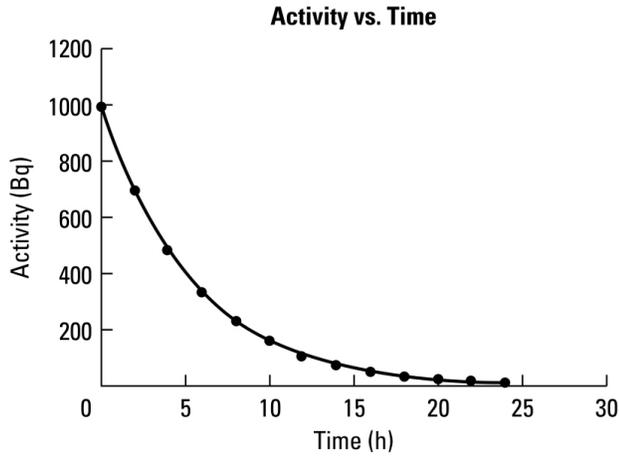
$$= -(1.98 \times 10^{-11} \text{ s}^{-1}) (5.1 \times 10^{15}) \left(\frac{1}{2} \right)^{5000/1600}$$

$$= -1.1 \times 10^4 \text{ Bq}$$

Paraphrase

After 5000 years, the activity of the dials will have dropped by about a factor of 10 to about $1.1 \times 10^4 \text{ Bq}$.

30.



- (a) By inspection, when $t = 5$ h, the activity of the sample is 400 Bq.
 (b) To estimate the half-life, note how long it takes the activity to drop from 400 Bq to 200 Bq. At $t = 5$ h, the activity is 400 Bq. At $t = 9$ h, the activity is 200 Bq. It has taken approximately 4 h for the activity to drop by a factor of one-half, so the half-life is approximately 4 h.

31. **Given**

Three helium-4 nuclei combine to form a carbon-12 nucleus.

Required

energy released (ΔE)

Analysis and Solution

Use the atomic mass data on p. 881 of the student text to calculate the net change in mass resulting from the reaction.

$$\begin{aligned} m_i &= 3m_{\text{He}} \\ &= 3(4.002\,603\text{ u}) \\ &= 12.007\,809\text{ u} \end{aligned}$$

$$\begin{aligned} m_f &= m_{\text{C}} \\ &= 12.000\,000\text{ u} \end{aligned}$$

$$\begin{aligned} m_i - m_f &= 12.007\,809\text{ u} - 12.000\,000\text{ u} \\ &= 0.007\,809\text{ u} \end{aligned}$$

Use mass-energy equivalence to calculate the energy released.

1 u is equivalent to 931.5 MeV, so

$$\begin{aligned} \Delta E &= 0.007\,809\text{ u} \times \frac{931.5\text{ MeV}}{1\text{ u}} \\ &= 7.274\text{ MeV} \end{aligned}$$

Paraphrase

The triple- α process that converts helium into carbon releases 7.274 MeV of energy.

32. (a) The key advantage of polonium over a chemical fuel is that you will obtain a much higher energy yield per unit mass by using nuclear decay rather than energy conversion through chemical means.

(b) Given

$$P = 20 \text{ W}$$

$$t = 14.5 \text{ years}$$

$$\text{efficiency} = 15\%$$

fuel = polonium-208

polonium-208 decays into lead-204

$$\lambda = 7.57 \times 10^{-9} \text{ s}^{-1}$$

$$t_{1/2} = 2.9 \text{ years}$$

Required

mass of polonium-208 (m)

Analysis and Solution

Since ${}^{208}_{84}\text{Po}$ decays into ${}^{204}_{82}\text{Pb}$, the pattern is that of an alpha decay—the atomic mass numbers and the atomic numbers of the two elements differ by a helium nucleus.

The energy released is equivalent to the difference between the mass of the parent atom and the total mass of the products.

$$\Delta m = m_{\text{parent}} - m_{\text{products}}$$

Look up the masses of the elements on p. 881 of the student text.

$$m_{\text{Po}} = 207.981\,246 \text{ u}$$

$$m_{\text{Pb}} = 203.973\,044 \text{ u}$$

$$m_{\alpha} = 4.002\,603 \text{ u}$$

$$\begin{aligned}\Delta m &= m_{\text{Po}} - (m_{\text{Pb}} + m_{\alpha}) \\ &= 207.981\,246 \text{ u} - 203.973\,044 \text{ u} - 4.002\,603 \text{ u} \\ &= 0.005\,599 \text{ u}\end{aligned}$$

Determine the energy released during the alpha decay of polonium into lead using the mass-energy equivalence, where $1 \text{ u} = 931.5 \text{ MeV}$.

$$\begin{aligned}\Delta E &= 0.005\,599 \cancel{\text{u}} \times \frac{931.5 \text{ MeV}}{1 \cancel{\text{u}}} \\ &= 5.215 \text{ MeV}\end{aligned}$$

Convert this value to joules.

$$\begin{aligned}\Delta E &= (5.215 \times 10^6 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}} \right) \\ &= 8.34 \times 10^{-13} \text{ J}\end{aligned}$$

Calculate the total number of decays needed to produce 20 W of electrical power. Remember that the energy conversion is only 15% efficient!

Since you need 20 J/s of electrical energy, you will need $\frac{20 \text{ J/s}}{0.15} = 133 \text{ J/s}$ of

thermal energy produced by the decay of polonium. Since each decay releases $8.34 \times 10^{-13} \text{ J}$, you require

$$\frac{133 \cancel{\text{J}}}{8.34 \times 10^{-13} \cancel{\text{J}}} = 1.59 \times 10^{14} \text{ decays per second}$$

This value is the same as the activity of the polonium fuel after 14.5 years.

The activity is $A = -\lambda N$, where λ is the decay constant, $7.57 \times 10^{-9} \text{ s}^{-1}$, and N is the total number of polonium atoms present. Solve for N :

$$\begin{aligned} N &= -\frac{A}{\lambda} \\ &= -\frac{1.59 \times 10^{14} \text{ s}^{-1}}{7.57 \times 10^{-9} \text{ s}^{-1}} \\ &= -2.10 \times 10^{22} \end{aligned}$$

Since you require 2.10×10^{22} atoms of polonium to be present after 14.5 years to ensure an electrical power output of 20 W, you must start with more polonium. To determine the original number of polonium atoms required, N_0 , use the

equation $N = N_0 \left(\frac{1}{2} \right)^{t/T_{1/2}}$.

$$\begin{aligned} N_0 &= \frac{N}{\left(\frac{1}{2} \right)^{t/T_{1/2}}} \\ &= \frac{2.10 \times 10^{22}}{\left(\frac{1}{2} \right)^{14.5/2.9}} \\ &= 6.72 \times 10^{23} \end{aligned}$$

Divide by Avogadro's number to find the number of moles of polonium required:

$$\frac{6.72 \times 10^{23} \text{ atoms}}{6.02 \times 10^{23} \frac{\text{atoms}}{\text{mol}}} = 1.12 \text{ mol}$$

Since one mole of polonium has a mass of 208 g, the mass of polonium required

$$\text{is } 1.12 \text{ mol} \times 208 \frac{\text{g}}{\text{mol}} = 233 \text{ g} = 0.23 \text{ kg}$$

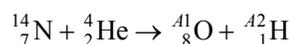
Paraphrase

Powering the space probe for 14.5 years requires only 0.23 kg of polonium. This mass is much less than that required if chemical fuels were used.

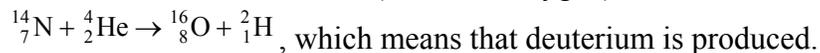
Extensions

- 33.** Start with the atomic notation for all the nuclei involved. Make sure that charge and atomic mass number are conserved. Remember that most elements have isotopes. To determine which reaction is more likely, look at the mass defects (differences between initial and final masses). The smallest mass defect will also represent the reaction that requires the least energy. This reaction will be the more probable one.

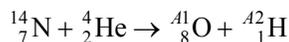
Solution 1



$A1$ and $A2$ need to be adjusted so that atomic mass number is conserved. A simple solution is to leave $A1$ as 16 ("normal" oxygen). Then, $A2 = 2$. The reaction is



Solution 2



If $A1 = 17$ and $A2 = 1$, then the reaction is ${}^{14}_7\text{N} + {}^4_2\text{He} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H}$.

To assess which reaction is more probable, note the following, where Mass 1 and Mass 2 refer to the two possible products given for Solutions 1 and 2, respectively.

Solution	Mass N-14	Mass He-4	Mass 1	Mass 2	Δm (u)	ΔE (MeV)
1	14.003 074	4.002 603	15.994 915	2.014 102	0.003 340	3.11
2	14.003 074	4.002 603	16.999 132	1.007 825	0.001 280	1.19

The more likely reaction is the one in Solution 2 because it is the one with the smaller net change of mass. An easy way to test this solution is to look for gamma rays. O-17 will be in an excited (nuclear) state and will decay via gamma-ray emission. Another test is to measure the charge-to-mass ratio for the “protons” created in the collision. In Solution 1, the “protons” are deuterons and the charge-to-mass ratio is one-half the ratio for normal protons (Solution 2).

Paraphrase

Two different isotopes of oxygen and hydrogen could be produced in this reaction.

The more likely reaction is the reaction, ${}^{14}_7\text{N} + {}^4_2\text{He} \rightarrow {}^{17}_8\text{O} + {}^1_1\text{H}$.

34. The two most serious radioactive isotopes released in a nuclear blast are strontium-90 and cesium-137. Both of these isotopes have half-lives of approximately 30 years. They can be spread globally by high-altitude winds, eventually be taken up by plant matter through rainfall, and then enter the human food chain. The build-up of either of these isotopes in the human body is a serious medical concern and is directly linked to various forms of cancer.
35. Nucleosynthesis is the process in which heavier elements are produced from lighter elements in the central regions of stars. The two main processes responsible are nuclear fusion and the neutron capture process that occurs during a supernova explosion.

Some of the fusion reactions are:

proton + proton \rightarrow He (proton-proton chain)
He + He + He \rightarrow C (triple alpha)
He + C \rightarrow O
C + p \rightarrow N (Other p and C combinations yield many elements.)
O + O \rightarrow Si (All other elements between O and Si are produced by various combinations of lighter nuclei.)
${}^{28}\text{Si} + {}^{28}\text{Si} \rightarrow {}^{56}\text{Ni} + \gamma$
${}^{56}\text{Ni} \rightarrow {}^{56}\text{Co} + {}^0_1\beta + \nu$
${}^{56}\text{Co} \rightarrow {}^{56}\text{Fe} + {}^0_1\beta + \nu$

When iron production is reached (assuming the star is massive enough), any reaction after this point absorbs rather than releases energy. Smaller stars are unable to complete these sequences because reactions beyond the proton-proton chain require

much higher temperatures. These temperatures can only be achieved if the mass of the star is high enough to compress the gas at its core to sufficiently high pressure and temperature. Our Sun will likely only reach the triple-alpha stage, where the core temperature climbs from 14×10^6 K to about 100×10^6 K.