

Pearson Physics Level 30
Unit VII Electromagnetic Radiation: Chapter 14
Solutions

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Concept Check

The colours of the stars are a first indication of the temperatures of their surfaces. The bright red star, Aldebaran (in Taurus), has a surface temperature of about 4000 K, whereas the bright blue star, Vega (high overhead in the summer sky), has a surface temperature of nearly 10 000 K.

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Concept Check

From the simple form of Planck's formula, $E = hf$, substitute $f = \frac{c}{\lambda}$. Combining these equations results in $E = \frac{hc}{\lambda}$.

Example 14.1 Practice Problems

1. **Given**

$$f = 4.00 \times 10^{14} \text{ Hz}$$

Required

photon energy (E)

Analysis and Solution

To find the energy of the photon, substitute into Planck's formula.

$$E = nhf$$

$$\begin{aligned} E &= (1)(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}})(4.00 \times 10^{14} \cancel{\text{s}}^{-1}) \\ &= 2.65 \times 10^{-19} \text{ J} \end{aligned}$$

Paraphrase

A photon of frequency 4.00×10^{14} Hz has an energy of 2.65×10^{-19} J.

2. **Given**

$$\lambda = 555 \text{ nm} = 5.55 \times 10^{-7} \text{ m}$$

Required

photon energy (E)

Analysis and Solution

To find the energy of the photon from its wavelength, substitute into Planck's formula.

$$E = nhf = \frac{nhc}{\lambda}$$

$$(1) \left(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}} \right) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)$$

$$= \frac{\quad}{5.55 \times 10^{-7} \cancel{\text{m}}}$$

$$= 3.58 \times 10^{-19} \text{ J}$$

Paraphrase

A photon of wavelength 555 nm has an energy of $3.58 \times 10^{-19} \text{ J}$.

3. Given

15.0 eV

Required

15.0 eV in joules

Analysis and Solution

If $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$, then

$$(15.0 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}} \right) = 2.40 \times 10^{-18} \text{ J}$$

Paraphrase

15.0 eV is the same as $2.40 \times 10^{-18} \text{ J}$.

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Example 14.2 Practice Problems

1. Given

$$\lambda = 10 \text{ nm} = 1.0 \times 10^{-8} \text{ m}$$

Required

frequency (f)

Analysis and Solution

To find frequency, substitute into the universal wave equation.

$$f = \frac{c}{\lambda}$$

$$= \frac{3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{1.0 \times 10^{-8} \cancel{\text{m}}}$$

$$= 3.0 \times 10^{16} \text{ Hz}$$

Paraphrase

A 10-nm photon has a frequency of $3.0 \times 10^{16} \text{ Hz}$. This photon is a “hard” or deep UV photon.

2. Given

$$\lambda = 10 \text{ nm}$$

$$f = 3.0 \times 10^{16} \text{ Hz (from question 1)}$$

Required

energy (E)

Analysis and Solution

Use $E = nhf$ and the given value for f .

$$E = (1)(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.0 \times 10^{16} \text{ Hz})$$

$$= 2.0 \times 10^{-17} \text{ J}$$

Paraphrase

A 10-nm photon has an energy of $2.0 \times 10^{-17} \text{ J}$.

3. Given

$$\lambda = 550 \text{ nm} = 5.50 \times 10^{-7} \text{ m}$$

$$E = 10 \text{ J}$$

Required

number of photons (n)

Analysis and Solution

Find f for the photon.

$$f = \frac{c}{\lambda}$$

$$= \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{5.50 \times 10^{-7} \text{ m}}$$

$$= 5.45 \times 10^{14} \text{ Hz}$$

Use $E = nhf$ to find the energy per photon, and divide the total energy (10 J) by this number.

$$E = (6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}})(5.45 \times 10^{14} \cancel{\text{ s}^{-1}})$$

$$= 3.61 \times 10^{-19} \text{ J}$$

$$E = nhf$$

$$n = \frac{E}{hf}$$

$$= \frac{10 \cancel{\text{ J}}}{3.61 \times 10^{-19} \cancel{\text{ J}}}$$

$$= 2.8 \times 10^{19}$$

Paraphrase

You need 2.8×10^{19} photons of green light (550 nm) to deliver 10 J of energy.

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Example 14.3 Practice Problems

1. Given

$$n = 1000$$

$$\lambda = 400 \text{ nm} = 4.00 \times 10^{-7} \text{ m}$$

Required

energy (E)

Analysis and Solution

Substitute into Planck's formula.

$$E = nhf \text{ where } f = \frac{c}{\lambda}$$

$$E = \frac{nhc}{\lambda}$$

$$= \frac{(1000)(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{4.00 \times 10^{-7} \cancel{\text{m}}}$$

$$= 4.97 \times 10^{-16} \text{ J}$$

Paraphrase

The beam delivers 4.97×10^{-16} J of energy.

2. Given

$$P = 10 \text{ W} = 10 \text{ J/s}$$

$$\lambda = 400 \text{ nm} = 4.00 \times 10^{-7} \text{ m}$$

Required

number of photons per second (n/s)

Analysis and Solution

Use $E = nhf = nh \left(\frac{c}{\lambda} \right)$, where $E = 10 \text{ J}$.

Solve for n using the equation

$$n = \frac{E\lambda}{hc}$$

$$n = \frac{\left(10 \frac{\cancel{\text{J}}}{\cancel{\text{s}}} \right) (4.00 \times 10^{-7} \cancel{\text{m}})}{(6.63 \times 10^{-34} \cancel{\text{J}} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}$$

$$= 2.0 \times 10^{19} \text{ photons/s}$$

Paraphrase

You need 2.0×10^{19} 400-nm photons per second to deliver 10 W of power.

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14.1 Check and Reflect

Knowledge

1. Given

$$\lambda = 450 \text{ nm} = 4.50 \times 10^{-7} \text{ m}$$

Required

energy of the photon (E)

Analysis and Solution

Use the equation $E = nhf$, where $n = 1$.

$$\begin{aligned}
 E &= nhf \\
 &= \frac{hc}{\lambda} \\
 &= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{4.50 \times 10^{-7} \cancel{\text{m}}} \\
 &= 4.42 \times 10^{-19} \text{ J}
 \end{aligned}$$

Paraphrase

The energy of the 450-nm photon is $4.42 \times 10^{-19} \text{ J}$.

2. Given

$$E = 15.0 \text{ eV}$$

Required

wavelength (λ)

Analysis and Solution

Use the equation $E = nhf$, where $f = \frac{c}{\lambda}$. Recall that $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.

$$\begin{aligned}
 E &= nh \left(\frac{c}{\lambda} \right) \\
 \lambda &= \frac{nhc}{E} \\
 &= \frac{(1)(6.63 \times 10^{-34} \cancel{\text{J}} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{(15.0 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\cancel{\text{J}}}{\cancel{\text{eV}}} \right)} \\
 &= 8.29 \times 10^{-8} \text{ m}
 \end{aligned}$$

Paraphrase

The wavelength of a 15.0-eV photon is $8.29 \times 10^{-8} \text{ m}$.

3. Given

$$\lambda = 300 \text{ nm} = 3.00 \times 10^{-7} \text{ m}$$

$$\lambda = 600 \text{ nm} = 6.00 \times 10^{-7} \text{ m}$$

Required

photon energy (E)

Analysis and Solution

Use the equation $E = nhf = \frac{nhc}{\lambda}$, where $n = 1$.

$$\begin{aligned}
 E_{300} &= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{3.00 \times 10^{-7} \cancel{\text{m}}} \\
 &= 6.63 \times 10^{-19} \text{ J}
 \end{aligned}$$

$$E_{600} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{6.00 \times 10^{-7} \cancel{\text{m}}}$$

$$= 3.32 \times 10^{-19} \text{ J}$$

Paraphrase

The 300-nm photon has an energy of $6.63 \times 10^{-19} \text{ J}$ and the 600-nm photon has an energy of $3.32 \times 10^{-19} \text{ J}$. The 300-nm photon is therefore twice as energetic as the 600-nm photon, or $E_{300} = 2E_{600}$.

4. (a) Given

$$E = 100 \text{ keV} = 1.00 \times 10^5 \text{ eV}$$

Required

frequency (f)

Analysis and Solution

Convert energy from keV to J.

$$100 \text{ keV} = (1.00 \times 10^5 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}} \right)$$

$$= 1.60 \times 10^{-14} \text{ J}$$

Use the equation $E = nhf$, where $n = 1$.

$$f = \frac{E}{h}$$

$$= \frac{1.60 \times 10^{-14} \cancel{\text{J}}}{6.63 \times 10^{-34} \frac{\cancel{\text{J}}}{\text{s}}}$$

$$= 2.41 \times 10^{19} \text{ Hz}$$

Paraphrase

The photon has a frequency of $2.41 \times 10^{19} \text{ Hz}$.

(b) From Figure 14.6, this photon is a gamma ray.

Applications

5. Given

$$P = 100 \text{ W} = 100 \text{ J/s}$$

$$\lambda = 550 \text{ nm} = 5.50 \times 10^{-7} \text{ m}$$

$$\Delta t = 10.0 \text{ s}$$

Required

number of photons emitted (n)

Analysis and Solution

$$P = \frac{E}{\Delta t} \text{ and } E = nhf = nh \left(\frac{c}{\lambda} \right), \text{ so } E = P\Delta t = \frac{nhc}{\lambda}. \text{ Solve for } n.$$

$$\begin{aligned}
 n &= \frac{P\lambda\Delta t}{hc} \\
 &= \frac{\left(100 \frac{\text{J}}{\text{s}}\right)\left(5.50 \times 10^{-7} \text{ m}\right)\left(10.0 \text{ s}\right)}{\left(6.63 \times 10^{-34} \frac{\text{J}}{\text{s}}\right)\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} \\
 &= 2.77 \times 10^{21}
 \end{aligned}$$

Paraphrase

In 10.0 s, a 100-W light bulb emits 2.77×10^{21} photons.

6. Given

$$P_{\text{Sun}} = 1400 \text{ W/m}^2$$

$$\lambda_{\text{ave}} = 700 \text{ nm} = 7.00 \times 10^{-7} \text{ m}$$

Required

number of photons per second per square metre (n)

Analysis and Solution

$$E = \frac{P}{\Delta t}$$

$$\frac{P}{\text{Area}} = \frac{1400 \text{ W}}{1 \text{ m}^2}$$

$$= 1.400 \times 10^3 \text{ W/m}^2$$

Let $\Delta t = 1 \text{ s}$ and area = 1 m^2 .

$$E = 1.400 \times 10^3 \text{ J}$$

$$= nhf$$

$$= nh\left(\frac{c}{\lambda}\right)$$

Solve for n .

$$n = \frac{E\lambda}{hc}$$

$$\begin{aligned}
 &= \frac{(1.400 \times 10^3 \text{ J})(7.00 \times 10^{-7} \text{ m})}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} \\
 &= 4.93 \times 10^{21}
 \end{aligned}$$

$$= 4.93 \times 10^{21}$$

Paraphrase

A 1-m^2 area receives approximately 4.93×10^{21} photons each second.

Extensions

7. Given

$$n = 10\,000 \text{ photons/s}$$

$$d_2 = 10d_1$$

Required

number of photons (n)

Analysis and Solution

The light will spread out over a sphere. $A = 4\pi r^2$ = surface area of a sphere. Since you are increasing r by a factor of 10, the area over which the photons are spread increases

by 100. The number of photons emitted by the star remains the same but is spread over a larger area. So, the number of photons received each second will drop by a corresponding factor of 100 or $\frac{10\,000}{100} = 100$ photons each second.

Paraphrase

You will receive 100 photons per second from the more distant star.

8. Given

$$P = 100 \text{ W}$$

$$n = 500 \text{ photons}$$

Required

distance (d)

average wavelength (λ_{ave})

area of light bulb (A)

Analysis and Solution

The choice for wavelength will vary. Use $\lambda = 600 \text{ nm}$.

A typical light bulb has a radius of 0.030 m , so let $r_i = 0.030 \text{ m}$.

$$\text{The number of photons per square metre} = \frac{\text{number emitted}}{4\pi r_i^2}$$

To find the number of photons emitted by the light bulb, use the equation

$$E = nhf$$

$$n = \frac{E}{hf} = \frac{E\lambda}{hc}$$

$$n = \frac{(100 \cancel{\text{ J}})(6.00 \times 10^{-7} \cancel{\text{ m}})}{(6.63 \times 10^{-34} \cancel{\text{ J}} \cdot \cancel{\text{ s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{ m}}}{\cancel{\text{ s}}} \right)}$$

$$= 3.0 \times 10^{20}$$

This value gives the density of photons, σ .

$$\begin{aligned} \text{The density of the photons} &= \frac{n}{4\pi r^2} \\ &= \frac{3.0 \times 10^{20} \text{ photons}}{4\pi(0.030 \text{ m})^2} \\ &= 2.7 \times 10^{22} \text{ photons/m}^2 \end{aligned}$$

The area of the human eye = πr_{pupil}^2

The number of photons received by the eye = (density of photons)(area of eye):

$$N = \sigma \pi r_{\text{pupil}}^2$$

A quick Web search reveals that the radius of the pupil (r_{pupil}) is 3.5 mm in the dark.

Use a value for a *dark* adapted eye for a bulb that is just discernible.

$$\begin{aligned} N &= \sigma \pi r_{\text{pupil}}^2 \\ &= \left(2.7 \times 10^{22} \frac{\text{photons}}{\text{m}^2} \right) \pi \left(3.5 \times 10^{-3} \text{ m} \right)^2 \\ &= 1.0 \times 10^{18} \text{ photons} \end{aligned}$$

Once you know how many photons are entering the eye when it is at the surface of the bulb, move to a distance r_f , where $\frac{N}{4\pi r_f^2} = 500$ photons. Use the same reasoning used in question 7 to determine the distance at which your eye would detect 500 photons. N varies directly as the square of the distance (see problem 7), so

$$\begin{aligned}
 500 &= N \times \left(\frac{r_i}{r_f} \right)^2 \\
 r_f^2 &= \frac{Nr_i^2}{500} \\
 r_f &= \sqrt{\frac{Nr_i^2}{500}} \\
 &= \sqrt{\frac{(1.0 \times 10^{18} \text{ photons})(0.030 \text{ m})^2}{500 \text{ photons}}} \\
 &= \sqrt{\frac{1.0 \times 10^{18}}{500}} \times 0.030 \text{ m} \\
 &= 1.3 \times 10^6 \text{ m} \\
 &= 1300 \text{ km}
 \end{aligned}$$

Paraphrase

This solution suggests that you could see a 100-W light bulb from more than 1300 km away! This result would only be possible in a completely dark environment, with no absorbing dust or haze. A more probable figure is in the tens of kilometres because the night sky is never completely dark. Another complication is the curvature of Earth, which becomes a factor for distances longer than 10 km. This problem raises many questions.

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Concept Check (top)

Photoemission will occur only if the incident photon has an energy greater than the work function for the metal. This relationship can be expressed as $E_{\text{photon}} = hf > W$.

Concept Check (bottom)

The collector plate is given a positive charge so that it will attract photoelectrons that have been knocked free from the metal surface by incident photons. Even without a charge on the collector plate, some photoelectrons would migrate down the tube and reach the collector, establishing a very weak electric current.

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Example 14.4 Practice Problems

1. Given

$$E = 5.3 \times 10^{-19} \text{ J}$$

Requiredstopping potential (V_{stopping})**Analysis and Solution**

$$E_{k_{\text{max}}} = qV_{\text{stopping}}$$

$$\begin{aligned} V_{\text{stopping}} &= \frac{E_{k_{\text{max}}}}{q} \\ &= \frac{5.3 \times 10^{-19} \text{ J}}{1.60 \times 10^{-19} \text{ C}} \\ &= 3.3 \text{ J/C} \\ &= 3.3 \text{ V} \end{aligned}$$

Paraphrase

The stopping potential required is 3.3 V.

2. Given

$$E = 5.3 \times 10^{-19} \text{ J}$$

Required

$$5.3 \times 10^{-19} \text{ J in eV}$$

Analysis and Solution

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$5.3 \times 10^{-19} \cancel{\text{ J}} \times \frac{1 \text{ eV}}{1.60 \times 10^{-19} \cancel{\text{ J}}} = 3.3 \text{ eV}$$

Paraphrase $5.3 \times 10^{-19} \text{ J}$ is equal to 3.3 eV.**3. Given**

$$V_{\text{stopping}} = 3.1 \text{ V}$$

Requiredmaximum kinetic energy of electrons ($E_{k_{\text{max}}}$)**Analysis and Solution**

$$\begin{aligned} E_{k_{\text{max}}} &= qV_{\text{stopping}} \\ &= (1.60 \times 10^{-19} \text{ C})(3.1 \text{ V}) \\ &= 5.0 \times 10^{-19} \text{ J} \\ &\text{or} \\ &= (1 \text{ e}^-)(3.1 \text{ V}) \\ &= 3.1 \text{ eV} \end{aligned}$$

ParaphraseThe maximum kinetic energy of the electrons is 3.1 eV or $5.0 \times 10^{-19} \text{ J}$.**Concept Check**

Relate the general expression $hf = qV_{\text{stopping}} + W$ to Planck's constant, h , by noting that qV_{stopping} is zero when f is the threshold frequency, f_0 . This means that

$$\begin{aligned}
 h &= \frac{qV_{\text{stopping}} + W}{f_0} \\
 &= \frac{qV_{\text{stopping}}}{f_0} + \frac{W}{f_0} \\
 &= \frac{W}{f_0}
 \end{aligned}$$

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Example 14.5 Practice Problems

1. Given

$$\lambda = 480 \text{ nm} = 4.80 \times 10^{-7} \text{ m}$$

Required

work function (W)

Analysis and Solution

$$W = hf_0$$

$$= \frac{hc}{\lambda_0}$$

$$= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{4.80 \times 10^{-7} \cancel{\text{m}}}$$

$$= 4.14 \times 10^{-19} \cancel{\text{J}} \times \frac{1 \text{ eV}}{1.60 \times 10^{-19} \cancel{\text{J}}}$$

$$= 2.59 \text{ eV}$$

Paraphrase and Verify

The metal requires photons of at least 2.59 eV for photoemission to occur. This value is reasonable given the data in Table 14.1.

2. Given

$$\lambda = 410 \text{ nm} = 4.10 \times 10^{-7} \text{ m}$$

$$W = 2.10 \text{ eV}$$

Required

kinetic energy of the photoelectron (E_k)

Analysis and Solution

$$\begin{aligned}
E_k &= E_{\text{photon}} - W \\
&= hf - W \\
&= \frac{hc}{\lambda} - W \\
&= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{4.10 \times 10^{-7} \cancel{\text{m}}} - (2.10 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}} \right) \\
&= 4.851 \times 10^{-19} \text{ J} - 3.360 \times 10^{-19} \text{ J} \\
&= 1.491 \times 10^{-19} \cancel{\text{J}} \times \frac{1 \text{ eV}}{1.60 \times 10^{-19} \cancel{\text{J}}} \\
&= 0.932 \text{ eV}
\end{aligned}$$

Paraphrase

The photoelectron leaves with a kinetic energy of $1.49 \times 10^{-19} \text{ J}$ or 0.932 eV , which represents the energy left after liberating the photoelectron.

Student Book page 719**Example 14.6 Practice Problems****1. Given**

$$E_k = 2.1 \text{ eV}$$

Required

speed of the photoelectron (v)

Analysis and Solution

Convert energy units from electron volts to joules.

$$\begin{aligned}
E_k &= (2.1 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}} \right) \\
&= 3.36 \times 10^{-19} \text{ J}
\end{aligned}$$

$$E_k = \frac{1}{2} mv^2$$

$$v = \sqrt{\frac{2E_k}{m}}$$

An electron has a mass of $9.11 \times 10^{-31} \text{ kg}$.

$$\begin{aligned}
v &= \sqrt{\frac{2(3.36 \times 10^{-19} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}} \\
&= 8.6 \times 10^5 \text{ m/s}
\end{aligned}$$

Paraphrase

The electron is moving with a speed of $8.6 \times 10^5 \text{ m/s}$ or 860 km/s .

2. Given

$$W = 2.10 \text{ eV (from Table 14.1)}$$

$$\lambda = 400 \text{ nm} = 4.00 \times 10^{-7} \text{ m}$$

Required

kinetic energy of the photoelectron (E_k)

Analysis and Solution

$$\begin{aligned}
 E_k &= E_{\text{photon}} - W \\
 &= \frac{hc}{\lambda} - W \\
 &= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)}{(4.00 \times 10^{-7} \text{ m}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right)} - 2.10 \text{ eV} \\
 &= 3.108 \text{ eV} - 2.10 \text{ eV} \\
 &= 1.01 \text{ eV}
 \end{aligned}$$

Paraphrase

The photoelectron has a kinetic energy of 1.01 eV.

3. Given

$$E_k = 1.01 \text{ eV} \times 1.61 \times 10^{-19} \frac{\text{J}}{\text{eV}} = 1.61 \times 10^{-19} \text{ J}$$

Required

maximum speed of the photoelectron (v)

Analysis and Solution

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$E_k = \frac{1}{2} m_e v^2$$

$$v = \sqrt{\frac{2E_k}{m_e}}$$

$$= \sqrt{\frac{2(1.61 \times 10^{-19} \text{ J})}{9.11 \times 10^{-31} \text{ kg}}}$$

$$= 5.95 \times 10^5 \text{ m/s}$$

Paraphrase

The photoelectron is moving with a maximum speed of $5.95 \times 10^5 \text{ m/s}$.

Student Book page 720**14.2 Check and Reflect****Knowledge****1. Given**

$$\lambda = 400 \text{ nm} = 4.00 \times 10^{-7} \text{ m}$$

Required

energy of the photon, in eV (E)

Analysis and Solution

$$E = nhf, \text{ where } n = 1$$

$$= \frac{hc}{\lambda}$$

$$= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{4.00 \times 10^{-7} \cancel{\text{m}}}$$

$$= \frac{4.97 \times 10^{-19} \cancel{\text{J}}}{1.60 \times 10^{-19} \frac{\cancel{\text{J}}}{\text{eV}}}$$

$$= 3.11 \text{ eV}$$

Paraphrase

The energy of the 400-nm photon is 3.11 eV.

2. The work function represents the amount of energy required to remove an electron from the metal. The frequency of the photon is directly related to its energy by Planck's formula, $E = nhf$. So, the photon's frequency must be great enough to make the product of Planck's constant and the photon's frequency greater than or equal to the work function, such that $E = hf \geq W$.

3. Given

cadmium

Required

threshold frequency (f_0)

Analysis and Solution

From Table 14.1, the work function for cadmium is 4.07 eV.

$$W = hf_0$$

$$W = (4.07 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}} \right)$$

$$= 6.51 \times 10^{-19} \text{ J}$$

$$f_0 = \frac{W}{h}$$

$$f_0 = \frac{6.51 \times 10^{-19} \cancel{\text{J}}}{6.63 \times 10^{-34} \cancel{\text{J}} \cdot \text{s}}$$

$$= 9.82 \times 10^{14} \frac{1}{\text{s}}$$

$$= 9.82 \times 10^{14} \text{ Hz}$$

Paraphrase and Verify

The photon must have a frequency of 9.82×10^{14} Hz, which means that its wavelength is 305 nm—in the UV range. This answer makes sense because the work function is quite large and hence requires an energetic photon.

4. Given

cesium $W = 2.10 \text{ eV}$

$$\lambda = 500 \text{ nm} = 5.00 \times 10^{-7} \text{ m}$$

Required

to determine whether photoemission occurs

Analysis and Solution

The kinetic energy of a photoelectron is given by the equation $E_k = hf - W$.

Kinetic energy must be positive, so photoemission will occur if $hf - W > 0$ or $hf > W$.

$$\begin{aligned}W &= (2.10 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}} \right) \\&= 3.36 \times 10^{-19} \text{ J} \\E_k &= hf \\&= \frac{hc}{\lambda} \\&= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{5.00 \times 10^{-7} \cancel{\text{m}}} \\&= 3.98 \times 10^{-19} \text{ J}\end{aligned}$$

Therefore, $hf > W$ and photoemission occurs.

Paraphrase and Verify

Cesium has a relatively low work function, so it is reasonable that a blue-green photon of wavelength 500 nm can cause photoemission.

5. $E_{k_{\max}} = qV_{\text{stopping}}$

$$\begin{aligned}V_{\text{stopping}} &= \frac{E_{k_{\max}}}{q} \\&= \frac{(1.25 \cancel{\text{eV}}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\cancel{\text{eV}}} \right)}{1.60 \times 10^{-19} \text{ C}} \\&= 1.25 \text{ J/C} \\&= 1.25 \text{ V}\end{aligned}$$

6. The stopping potential of the electron varies directly as its maximum kinetic energy.

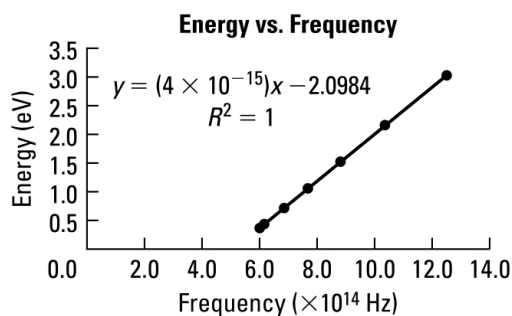
7. False. Increasing the intensity of the light source has no effect on the maximum kinetic energy of the electrons and hence on the stopping potential. The kinetic energy of electrons depends on their frequency, not on the intensity of the light.

Applications

8. Convert wavelength to frequency using the equation $f = \frac{c}{\lambda}$.

Wavelength (nm)	Frequency (Hz)	Kinetic Energy (eV)
500	$f = \frac{c}{\lambda}$ $= \frac{3.00 \times 10^8 \frac{\text{m}}{\text{s}}}{5.00 \times 10^{-7} \text{ m}}$ $= 6.00 \times 10^{14} \text{ Hz}$	0.36
490	$6.12 \times 10^{14} \text{ Hz}$	0.41
440	$6.82 \times 10^{14} \text{ Hz}$	0.70
390	$7.69 \times 10^{14} \text{ Hz}$	1.05
340	$8.82 \times 10^{14} \text{ Hz}$	1.52
290	$1.03 \times 10^{15} \text{ Hz}$	2.14
240	$1.25 \times 10^{15} \text{ Hz}$	3.025

The graph shows a straight-line relationship ($y = mx + b$) between f and E_k . This relationship helps show that $E_{k_{\max}} = hf - W$.



9. Given

the graph constructed in question 8

Required

slope (m)

Analysis and Solution

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

When comparing the equation $E_k = hf - W$ with the general equation $y = mx + b$, the slope of $E_k = hf - W$ should equal Planck's constant.

$$m = \frac{\Delta E_k}{\Delta f}$$

$$= \frac{(3.025 - 0.36) \text{ eV}}{(1.25 \times 10^{15} - 6.00 \times 10^{14}) \text{ s}^{-1}}$$

$$= 4.1 \times 10^{-15} \text{ eV} \cdot \text{s}$$

The slope is $4.1 \times 10^{-15} \text{ eV} \cdot \text{s}$.

Paraphrase

As expected, the slope of this graph is close to the value for Planck's constant.

10. To determine the metal used, find the work function, W , by noting where the y -intercept of the E_k - f graph occurs. From $E_k = hf - W$, when $f = 0$, $E_k = -W$.

A negative kinetic energy simply means that this amount of energy is required to just eject an electron from the metal. If you rearrange the equation to $E_k + W = 0$, then you can argue that the negative of the y -intercept is numerically equal to the work function.

The y -intercept of the graph is -2.1 eV. The y -intercept is the same as the work function. So, the metal in question has a work function of $-(-2.1 \text{ eV}) = 2.1$ eV. From Table 14.1, the metal with this work function value is cesium.

Extensions

11. According to the photon model of light, the energy of the photon arrives as a packet and is absorbed instantly (or very nearly so) by the metal. If the incident photon does not carry enough energy to dislodge an electron, the incident energy is quickly transferred by collisions between the electrons and atoms of the metal, thereby heating the metal. It is only when the incident photon delivers sufficient energy for the electron to overcome the work function for the metal's surface that emission will occur. The energy of the incident photon depends only on the photon's frequency, and not on the intensity of the incident light.
12. The photoelectric effect is at the heart of many common applications. Even though few of these applications consist of electrons being emitted from a metal surface into a vacuum chamber (as presented in most texts), they involve electrons and the absorption or emission of photons. One example is photovoltaic cells, which are made of semiconductors that convert photons from the Sun into electric current (electron flow). Photovoltaic cells are used in solar powered calculators and solar powered parking meters. Another common application of the photoelectric effect is the charge coupled device (CCD) that forms the heart of most digital cameras. Light strikes the pixels of the CCD and causes the release of electrons that accumulate on the pixel. The electrons create an electric potential that is eventually reconstructed as a digital picture.

13. **Given**

$$P = 2.0 \times 10^{-6} \text{ W}$$

$$A_{\text{beam}} = 1.0 \times 10^{-4} \text{ m}^2$$

$$E = W = 3.5 \text{ eV}$$

Required

time taken for an atom to absorb enough energy to emit an electron (Δt)

Analysis and Solution

The energy in the beam is spread over an area of $1.0 \times 10^{-4} \text{ m}^2$. An individual atom can only absorb a tiny portion of this energy. First calculate how much energy per second an individual atom receives.

The atomic radius is approximately 10^{-11} m . The surface area of an atom is therefore approximately 10^{-20} m^2 . Thus, any given atom only absorbs

$$\left(2.0 \times 10^{-6} \text{ W}\right) \left(\frac{10^{-20} \text{ m}^2}{10^{-4} \text{ m}^2}\right) = 2.0 \times 10^{-22} \text{ W} \text{ or } 2.0 \times 10^{-22} \text{ J/s.}$$

Determine how long it takes to absorb 3.5 eV.

Since $E = P_{\text{atom}}\Delta t$,

$$\begin{aligned}\Delta t &= \frac{E}{P_{\text{atom}}} \\ &= \frac{(3.5 \text{ eV}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}} \right)}{2.0 \times 10^{-22} \frac{\text{J}}{\text{s}}} \\ &= 2800 \text{ s or } 0.78 \text{ h}\end{aligned}$$

Paraphrase

According to classical theory, photoemission is a *slow* process: It would take minutes to hours for an atom to absorb enough energy to release an electron.

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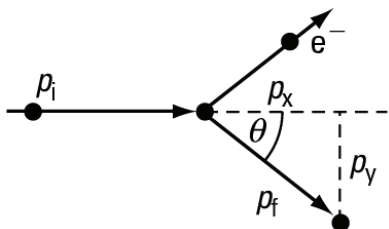
Concept Check

Momentum is inversely related to wavelength, so the 2-nm photon has the greatest momentum.

$$\begin{aligned}p_{500} &= \frac{h}{\lambda_{500}} \\ &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{5.00 \times 10^{-7} \text{ m}} \\ &= 1.33 \times 10^{-27} \text{ N}\cdot\text{s} \\ p_2 &= \frac{h}{\lambda_2} \\ &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{2 \times 10^{-9} \text{ m}} \\ &= 3.32 \times 10^{-25} \text{ N}\cdot\text{s} \\ p_2 &> p_{500}\end{aligned}$$

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Concept Check



1. Refer to the diagram above. Resolve p_f into x and y components:

$$p_{f_x} = p_f \cos \theta = \frac{h}{\lambda_f} \cos \theta$$

$$p_{f_y} = -p_f \sin \theta = -\frac{h}{\lambda_f} \sin \theta$$

2. From the original conditions, the net momentum in the y direction was zero. Hence, to conserve momentum in this direction, it follows that the electron must move in the y direction with equal and opposite momentum. To conserve momentum in the x direction, the sum of the x -direction momentum of the photon and the electron must equal the original momentum in the system. [**Note:** In a formal derivation of the Compton effect, you must use the relativistic concept of 4-momentum, which is beyond the scope of the high school curriculum.]
3. Because the scattered X ray has a longer wavelength, it has lost energy. The missing energy has been given to the electron, according to the law of conservation of energy. The electron's final energy can be written as:

$$\Delta E = h(f_f - f_i) = hc \left(\frac{1}{\lambda_f} - \frac{1}{\lambda_i} \right)$$

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Example 14.7 Practice Problems

1. **Given**

$$\lambda = 10 \text{ nm} = 10 \times 10^{-9} \text{ m}$$

Required

energy (E)

Analysis and Solution

$$E = hf = \frac{hc}{\lambda}$$

$$E = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}}) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right)}{10 \times 10^{-9} \cancel{\text{m}}}$$

$$= 2.0 \times 10^{-17} \text{ J}$$

Paraphrase

The energy of the 10-nm X ray is $2.0 \times 10^{-17} \text{ J}$.

2. **Given**

$$\lambda = 10 \text{ nm} = 10 \times 10^{-9} \text{ m}$$

Required

momentum (p)

Analysis and Solution

$$p = \frac{h}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{10 \times 10^{-9} \text{ m}}$$

$$= 6.6 \times 10^{-26} \text{ N} \cdot \text{s}$$

Paraphrase

The momentum of the X ray is $6.6 \times 10^{-26} \text{ N} \cdot \text{s}$.

3. Given

$$\lambda_i = 10 \text{ nm} = 10 \times 10^{-9} \text{ m}$$

$$\lambda_f = 11 \text{ nm} = 11 \times 10^{-9} \text{ m}$$

Required

change in energy (ΔE)

Analysis and Solution

$$E = \frac{hc}{\lambda}, \text{ so } \Delta E = hc \left(\frac{1}{\lambda_i} - \frac{1}{\lambda_f} \right)$$

$$\begin{aligned} \Delta E &= \left(6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}} \right) \left(3.00 \times 10^8 \frac{\cancel{\text{m}}}{\cancel{\text{s}}} \right) \left(\frac{1}{10 \times 10^{-9} \cancel{\text{m}}} - \frac{1}{11 \times 10^{-9} \cancel{\text{m}}} \right) \\ &= 1.8 \times 10^{-18} \text{ J} \end{aligned}$$

Paraphrase

The electron gains $1.8 \times 10^{-18} \text{ J}$ of energy.

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Example 14.8 Practice Problem

1. Given

$$\lambda_i = 0.010 \text{ nm} = 1.0 \times 10^{-11} \text{ m}$$

$$\theta = 90^\circ$$

Required

wavelength of scattered photon (λ_f)

Analysis and Solution

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{mc} (1 - \cos \theta)$$

$$\lambda_f = \lambda_i + \frac{h}{mc} (1 - \cos \theta)$$

$$\lambda_f = 1.0 \times 10^{-11} \text{ m} + \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)} (1 - 0)$$

$$= 1.2 \times 10^{-11} \text{ m}$$

$$= 0.012 \text{ nm}$$

Paraphrase

The wavelength of the scattered photon is 0.012 nm.

14.3 Check and Reflect

Knowledge

1. Given

$$\lambda = 500 \text{ nm} = 5.00 \times 10^{-7} \text{ m}$$

Required

momentum (p)

Analysis and Solution

$$\begin{aligned} p &= \frac{h}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{5.00 \times 10^{-7} \text{ m}} \\ &= 1.33 \times 10^{-27} \text{ N} \cdot \text{s} \end{aligned}$$

Paraphrase

A 500-nm photon has a momentum of $1.33 \times 10^{-27} \text{ N} \cdot \text{s}$.

2. Given

$$\lambda_A = 3\lambda_B$$

Required

greatest momentum (p)

Analysis and Solution

The momentum of a photon is inversely related to wavelength: $p = \frac{h}{\lambda}$.

$$\frac{p_A}{p_B} = \frac{\cancel{\lambda_A}}{\cancel{h}} = \frac{\lambda_B}{\lambda_A}$$

$$\text{But } \lambda_A = 3\lambda_B.$$

$$\frac{p_A}{p_B} = \frac{\cancel{\lambda_B}}{3\cancel{\lambda_B}}$$

$$p_A = \frac{1}{3} p_B$$

Paraphrase

Photon A has one-third the momentum of photon B.

3. Given

$$p = 6.00 \times 10^{-21} \text{ kg} \cdot \text{m/s}$$

Required

wavelength (λ)

energy (E)

Analysis and Solution

$$\begin{aligned} p &= \frac{h}{\lambda} \\ \lambda &= \frac{h}{p} \\ &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{6.00 \times 10^{-21} \text{ kg} \cdot \text{m/s}} \\ &= 1.11 \times 10^{-13} \text{ m} \\ E &= \frac{hc}{\lambda} \\ &= pc \\ &= (6.00 \times 10^{-21} \text{ kg} \cdot \text{m/s})(3.00 \times 10^8 \text{ m/s}) \\ &= 1.80 \times 10^{-12} \text{ J} \end{aligned}$$

Paraphrase

The photon has a wavelength of $1.11 \times 10^{-13} \text{ m}$ and an energy of $1.80 \times 10^{-12} \text{ J}$.

- The photon has a wavelength of $1.11 \times 10^{-13} \text{ m}$ or 0.000 111 nm. From Figure 14.6, this wavelength is in the “hard” X ray or gamma ray part of the electromagnetic spectrum.
- This statement is false. Physicists believe that the laws of conservation of momentum and of energy hold in all cases! They would rather give up a theory than either of these two fundamental laws.

Applications

6. Given

$$E = 100 \text{ keV}$$

Required

wavelength (λ)

Analysis and Solution

Convert 100 keV to joules: $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$.

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}, \text{ so } 100\,000 \text{ eV} = 1.60 \times 10^{-14} \text{ J}$$

Then solve for wavelength using the equation $E = \frac{hc}{\lambda}$.

$$\begin{aligned} \lambda &= \frac{hc}{E} \\ &= \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)}{1.60 \times 10^{-14} \text{ J}} \\ &= 1.24 \times 10^{-11} \text{ m} \\ &= 0.0124 \text{ nm} \end{aligned}$$

Paraphrase

An X ray of energy 100 keV has a wavelength of 0.0124 nm.

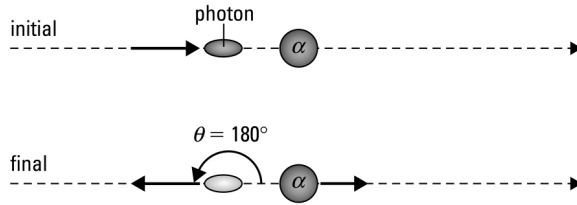
7. Given

$$\lambda = 0.010 \text{ nm} = 1.0 \times 10^{-11} \text{ m}$$

$$\theta_{\text{scatter}} = 180^\circ$$

Required

speed of helium nucleus (v)

Analysis and Solution

The mass of a helium nucleus is 6.6×10^{-27} kg.

Use Compton's scattering equation, $\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{mc}(1 - \cos \theta)$, to calculate the final wavelength of the X-ray photon.

$$\Delta\lambda = \lambda_f - \lambda_i = \frac{h}{mc}(1 - \cos \theta)$$

$$\lambda_f = \lambda_i + \frac{h}{mc}(1 - \cos \theta)$$

$$= 1.0 \times 10^{-11} \text{ m} + \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(6.6 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})}(1 - \cos 180^\circ)$$

$$= 1.000067 \times 10^{-11} \text{ m}$$

Use Planck's formula, $E = nhf$, where $n = 1$, to find the energy lost by the X ray.

$$\Delta E = h(f_f - f_i)$$

$$= hc \left(\frac{1}{\lambda_f} - \frac{1}{\lambda_i} \right)$$

$$= (6.63 \times 10^{-34} \text{ J}\cdot\text{s})(3.00 \times 10^8 \text{ m/s}) \left(\frac{1}{1.000067 \times 10^{-11} \text{ m}} - \frac{1}{1.0 \times 10^{-11} \text{ m}} \right)$$

$$= -1.33 \times 10^{-18} \text{ J}$$

From the law of conservation of energy, the energy lost by the X ray is the energy gained by the helium nucleus. Use the equation for kinetic energy, $\Delta E = \frac{1}{2}mv^2$, to find the final velocity of the helium nucleus.

$$\Delta E = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2\Delta E}{m_{\text{He}}}}$$

$$= \sqrt{\frac{2(1.33 \times 10^{-18} \text{ J})}{6.6 \times 10^{-27} \text{ kg}}}$$

$$= 2.0 \times 10^4 \text{ m/s}$$

Paraphrase

The X ray scatters off the helium nucleus and the helium nucleus recoils with a velocity of 2.0×10^4 m/s.

Extension

8. In order to see a small particle, you must choose a small wavelength. However, the smaller the wavelength, the greater is the momentum of the photon and, because of Compton scattering, the greater is the interaction between the photon and the particle. For this reason, the smaller the particle, the more difficult it is to see.

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Example 14.9 Practice Problem

1. **Given**

$$\lambda = 0.010 \text{ nm} = 1.0 \times 10^{-11} \text{ m}$$

Required

momentum (p)

Analysis and Solution

$$\begin{aligned} p &= \frac{h}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.0 \times 10^{-11} \text{ m}} \\ &= 6.6 \times 10^{-23} \text{ kg} \cdot \text{m/s} \end{aligned}$$

Paraphrase

An X-ray photon of wavelength 0.010 nm has a momentum of $6.6 \times 10^{-23} \text{ kg} \cdot \text{m/s}$.

Example 14.10 Practice Problems

1. **Given**

$$v = 1.0 \times 10^5 \text{ m/s}$$

Required

wavelength (λ)

Analysis and Solution

The mass of a proton is $1.67 \times 10^{-27} \text{ kg}$.
Assuming non-relativistic speeds,

$$\begin{aligned} p &= \frac{h}{\lambda} \\ \lambda &= \frac{h}{p} \\ &= \frac{h}{mv} \\ &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.67 \times 10^{-27} \text{ kg})(1.0 \times 10^5 \text{ m/s})} \\ &= 4.0 \times 10^{-12} \text{ m} \end{aligned}$$

Paraphrase

The de Broglie wavelength of the proton is $4.0 \times 10^{-12} \text{ m}$ or 0.0040 nm.

2. **Given**

$$\lambda = 420 \text{ nm} = 4.20 \times 10^{-7} \text{ m}$$

Requiredspeed (v)**Analysis and Solution**The mass of an electron is 9.11×10^{-31} kg .

Assuming non-relativistic speeds,

$$p = \frac{h}{\lambda} = mv$$

$$v = \frac{h}{m\lambda}$$

$$= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{(9.11 \times 10^{-31} \text{ kg})(4.20 \times 10^{-7} \text{ m})}$$

$$= 1.73 \times 10^3 \text{ m/s}$$

Paraphrase

An electron that has a de Broglie wavelength of 420 nm is moving with a speed of 1.73×10^3 m/s .

Student Book page 732**Example 14.12 Practice Problems****1. Given**

$$l = 1.0 \text{ nm}$$

Requiredmaximum wavelength (λ)**Analysis and Solution**

The wavelength and length of the box are related by the equation $\lambda_n = \frac{2l}{n}$.

For maximum wavelength, $n = 1$.

$$\lambda_1 = \frac{2l}{1}$$

$$= \frac{2(1.0 \text{ nm})}{1}$$

$$= 2.0 \text{ nm}$$

Paraphrase

The maximum (lowest) wavelength for the electron is 2.0 nm.

2. Given

$$\lambda = 2.0 \text{ nm} = 2.0 \times 10^{-9} \text{ m}$$

Requiredmomentum (p)**Analysis and Solution**

$$p = \frac{h}{\lambda}$$

$$= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{2.0 \times 10^{-9} \text{ m}}$$

$$= 3.3 \times 10^{-25} \text{ kg}\cdot\text{m/s}$$

Paraphrase

The momentum of the electron in the box is $3.3 \times 10^{-25} \text{ kg} \cdot \text{m/s}$.

3. Given

$$l = 1.0 \text{ nm}$$

Required

minimum energy (E_{min})

Analysis and Solution

For minimum kinetic energy, use the equation $E = \frac{p^2}{2m}$.

From question 2, the momentum of an electron in a box of length 1.0 nm is $3.3 \times 10^{-25} \text{ kg} \cdot \text{m/s}$.

The mass of an electron is $9.11 \times 10^{-31} \text{ kg}$.

$$\begin{aligned} E &= \frac{p^2}{2m} \\ &= \frac{(3.3 \times 10^{-25} \text{ kg} \cdot \text{m/s})^2}{2(9.11 \times 10^{-31} \text{ kg})} \\ &= 6.0 \times 10^{-20} \text{ J} \end{aligned}$$

Paraphrase

The minimum energy for an electron confined to a box 1.0 nm in length is $6.0 \times 10^{-20} \text{ J}$.

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Concept Check

The smaller the region, the smaller is the possible wavelength of the confined particle. If you reduce the size of the region from 10^{-10} m across to 10^{-15} m across, the wavelength of the particle would decrease by a factor of 10^5 . Momentum is inversely related to wavelength, so the momentum would increase by the same factor. Since energy varies directly as momentum squared, the energy would increase by a factor of 10^{10} . Typical energies on the atomic scale are measured between electron volts and a few tenths of electron volts, so energies on a nuclear scale are measured from MeV to GeV.

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14.4 Check and Reflect

Knowledge

1. Given

$$v = 20\,000 \text{ m/s}$$

Required

wavelength (λ)

Analysis and Solution

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$\begin{aligned}
 p &= \frac{h}{\lambda} \\
 \lambda &= \frac{h}{p} \\
 &= \frac{h}{mv} \\
 &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(20\,000 \text{ m/s})} \\
 &= 3.64 \times 10^{-8} \text{ m} \\
 &= 36.4 \text{ nm}
 \end{aligned}$$

Paraphrase

The electron's de Broglie wavelength is 36.4 nm.

2. Given

$$\lambda = 500 \text{ nm} = 5.00 \times 10^{-7} \text{ m}$$

Required

momentum (p)

Analysis and Solution

$$\begin{aligned}
 p &= \frac{h}{\lambda} \\
 &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{5.00 \times 10^{-7} \text{ m}} \\
 &= 1.33 \times 10^{-27} \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

Paraphrase

A 500-nm photon has a momentum of $1.33 \times 10^{-27} \text{ kg} \cdot \text{m/s}$.

3. Given

$$\Delta x = 1.0 \text{ nm} = 1.0 \times 10^{-9} \text{ m}$$

Required

the uncertainty in the momentum of the particle (Δp)

Analysis and Solution

$$\begin{aligned}
 \Delta x \Delta p &\geq \frac{h}{4\pi} \\
 \Delta p &\geq \frac{h}{4\pi} \left(\frac{1}{\Delta x} \right) \\
 &\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi} \left(\frac{1}{1.0 \times 10^{-9} \text{ m}} \right) \\
 &\geq 5.3 \times 10^{-26} \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

Paraphrase

The uncertainty in the momentum of the particle is $5.3 \times 10^{-26} \text{ kg} \cdot \text{m/s}$.

4. Given

$$\Delta x = 2.5 \times 10^{-12} \text{ m}$$

Required

the uncertainty in the electron's momentum (Δp)

Analysis and Solution

Treat the sphere's diameter as Δx .

$$\Delta p \Delta x \geq \frac{h}{4\pi}$$

$$\Delta p \geq \frac{h}{4\pi} \left(\frac{1}{\Delta x} \right)$$

$$\geq \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi} \left(\frac{1}{2.5 \times 10^{-12} \text{ m}} \right)$$

$$\geq 2.1 \times 10^{-23} \text{ kg} \cdot \text{m/s}$$

Paraphrase

The electron's momentum is no less than $2.1 \times 10^{-23} \text{ kg} \cdot \text{m/s}$. You could substitute this answer into the equation $E = \frac{p^2}{2m}$ to find the minimum energy of this electron.

Applications**5. (a) Given**

$$V = 21\,000 \text{ V}$$

Required

energy (E)

Analysis and Solution

The electric field does work on the electron ($W = qV$), which increases the electron's kinetic energy.

$$E_k = \frac{1}{2}mv^2$$

$$= qV$$

$$q = 1.60 \times 10^{-19} \text{ C}$$

$$E_k = (1.60 \times 10^{-19} \text{ C})(21\,000 \text{ V})$$

$$= 3.36 \times 10^{-15} \text{ J}$$

Paraphrase

An electron accelerated through a 21 000-V potential acquires an energy of $3.36 \times 10^{-15} \text{ J}$.

(b) Given

$$E_k = 3.36 \times 10^{-15} \text{ J}$$

Required

wavelength (λ)

Analysis and Solution

$$p = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p}$$

$$E_k = \frac{p^2}{2m}$$

$$p = \sqrt{2mE_k}$$

Therefore,

$$\lambda = \frac{h}{\sqrt{2mE_k}}$$
$$\lambda = \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(3.36 \times 10^{-15} \text{ J})}}$$
$$= 8.47 \times 10^{-12} \text{ m}$$

Paraphrase

An electron of this energy has a wavelength of $8.47 \times 10^{-12} \text{ m}$.

6. From $p = \frac{h}{\lambda}$ and $p = mv$, $\lambda = \frac{h}{mv}$. The ratio is

$$\frac{\lambda_p}{\lambda_e} = \frac{\cancel{h}}{m_p \cancel{v}} \cdot \frac{\cancel{h}}{m_e \cancel{v}}$$
$$= \frac{m_e}{m_p}$$

The wavelength varies inversely as the mass.

Substitute $m_e = 9.11 \times 10^{-31} \text{ kg}$ and $m_p = 1.67 \times 10^{-27} \text{ kg}$.

$$\frac{\lambda_p}{\lambda_e} = \frac{9.11 \times 10^{-31} \cancel{\text{kg}}}{1.67 \times 10^{-27} \cancel{\text{kg}}}$$
$$= 5.46 \times 10^{-4}$$

$$\text{or } \frac{\lambda_e}{\lambda_p} = 1.83 \times 10^3 \approx 2000$$

The wavelength of an electron moving with the same velocity as a proton is approximately 2000 times larger than the corresponding wavelength of the proton.

Extensions

7. No, a state of no motion at absolute zero is impossible in quantum physics. No motion implies that the particle has exactly zero momentum and hence no uncertainty in momentum. But, if you know where a particle is, then, by definition, you know that the uncertainty in its position must be no bigger than the size of the region it occupies.

The particle cannot have zero uncertainty in its momentum because $\Delta x \Delta p \geq \frac{h}{4\pi}$.

No motion is therefore an impossible state in quantum physics.

8. Given

$$E_n = \frac{n^2 h^2}{8ml^2}, \quad n = 1, 2, 3, \dots$$

Required

Derive the given expression.

Analysis and Solution

Use the concepts of (i) wavelength and standing waves in a one-dimensional box, (ii) the connection between wavelength and momentum, $p = \frac{h}{\lambda}$, and (iii) the connection

between E_k and p , where $E_k = \frac{p^2}{2m}$.

$$\begin{aligned}\lambda_n &= \frac{2l}{n} \\ p &= \frac{h}{\left(\frac{2l}{n}\right)} \\ &= \frac{nh}{2l} \\ E_k &= \frac{p^2}{2m} \\ &= \frac{\left(\frac{nh}{2l}\right)^2}{2m} \\ &= \frac{n^2 h^2}{8ml^2}\end{aligned}$$

Paraphrase

The energy of a particle of mass m trapped in a box of length l is quantized according to the expression $E_n = \frac{n^2 h^2}{8ml^2}$, where $n = 1, 2, 3, \dots$

Student Book page 740

14.5 Check and Reflect

Knowledge

1. The best choice is (a): The double-slit experiment illustrates the formation of nodes and antinodes—an interference pattern—which demonstrate that light has wave-like properties.
2. (a) False: Electrons can also form nodes and antinodes when beamed through a double slit.
(b) True: The Davisson-Germer experiment showed that electrons can form a series of nodes and antinodes when reflected off a nickel surface.

Applications

3. (a) This example illustrates the particle nature of a quantum because the electrons that hit the phosphor screen are localized in space and time.
(b) This example illustrates the wave nature of a quantum because the effect is that of wave interference.
(c) This example illustrates the particle nature of a quantum because light is localized in a small point at an instant of time.

Extension

4. If Planck's constant changed to $6.63 \text{ J}\cdot\text{s}$, then you would have a much larger wavelength! For example, if you have a mass of 66.3 kg and walk at a speed of 1.00 m/s , then your momentum is $66.3 \text{ kg}\cdot\text{m/s}$. Since $\lambda = \frac{h}{p}$, your wavelength would

$$\text{be } \lambda = \frac{6.63 \text{ J}\cdot\text{s}}{66.3 \text{ kg}\cdot\text{m/s}} = 0.100 \text{ m}.$$

You would diffract into a series of nodes and antinodes as you passed through the doorway!

Student Book pages 742–743

Chapter 14 Review

Knowledge

1. Classical physics predicted that a struck match would emit increasing amounts of light at shorter wavelengths. This effect would mean that a flame from a match would emit more blue light than red light, more UV light than blue light, more X rays than UV rays, and so on. This increasing amount of shorter-wavelength, and hence higher-energy, light would lead to the absurd prediction that a single match would incinerate the universe!
2. Planck's formula is $E = nhf$, where n is the number of photons. According to this equation, a photon of light has an energy that is equal to Planck's constant times the frequency of the light. When $n = 1$, Planck's formula becomes $E = hf$.

3. Given

$$\lambda = 450 \text{ nm} = 4.50 \times 10^{-7} \text{ m}$$

Required

energy (E)

Analysis and Solution

$$E = hf = \frac{hc}{\lambda}$$

$$= \frac{(6.63 \times 10^{-34} \text{ J}\cdot\text{s}) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}} \right)}{4.50 \times 10^{-7} \text{ m}}$$

$$= 4.42 \times 10^{-19} \text{ J}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$$

$$\frac{4.42 \times 10^{-19} \cancel{\text{J}}}{1.60 \times 10^{-19} \frac{\cancel{\text{J}}}{\text{eV}}} = 2.76 \text{ eV}$$

Paraphrase

A 450-nm photon has an energy of $4.42 \times 10^{-19} \text{ J}$ or 2.76 eV .

4. Energy varies directly as frequency and inversely as wavelength. If an X-ray photon has a wavelength 100 times smaller than a visible light photon, then its frequency, and hence its energy, will be 100 times greater than that of the visible light photon.
5. Heinrich Hertz is credited with discovering the photoelectric effect in 1887.

6. Einstein provided the explanation for the photoelectric effect by making the radical suggestion that light was behaving like a particle.

7. **Given**

$$f_0 = 6.0 \times 10^{14} \text{ Hz}$$

Required

work function (W)

Analysis and Solution

$$E_{k_{\max}} = hf - W$$

When the metal is illuminated with photons at the threshold frequency, electrons will just be emitted.

$$E_k \approx 0$$

$$hf - W = 0$$

$$W = hf$$

$$\begin{aligned} W &= (6.63 \times 10^{-34} \text{ J} \cdot \cancel{\text{s}})(6.0 \times 10^{14} \cancel{\text{ s}^{-1}}) \\ &= 4.0 \times 10^{-19} \text{ J} \\ &= \frac{4.0 \times 10^{-19} \cancel{\text{ J}}}{1.60 \times 10^{-19} \frac{\cancel{\text{ J}}}{\text{eV}}} \\ &= 2.5 \text{ eV} \end{aligned}$$

Paraphrase

If the threshold frequency for photoemission from a metal surface is 6.0×10^{14} Hz, then the work function for the metal is 4.0×10^{-19} J or 2.5 eV.

8. The Compton effect is the scattering of X rays when they interact with electrons or other particles. The particle model of light was able to predict and explain the small change in wavelength in the scattered X ray and the relationship between the change in wavelength and the scattering angle (not predicted by the classical theory). The momentum and energy of the scattered X ray changed exactly as if the X ray were a small particle undergoing an elastic collision with an electron. The Compton effect thereby provided critical evidence for the particle model of light.

9. **Given**

$$\lambda = 0.010 \text{ nm}$$

$$\theta = 90^\circ$$

Required

change in wavelength ($\Delta\lambda$)

Analysis and Solution

Use the equation $\Delta\lambda = \frac{h}{mc}(1 - \cos \theta)$.

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

$$\begin{aligned} \Delta\lambda &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^8 \text{ m/s})}(1 - \cos 90^\circ) \\ &= 2.4 \times 10^{-12} \text{ m} \\ &= 0.0024 \text{ nm} \end{aligned}$$

Paraphrase

The photon undergoes a wavelength increase of 0.0024 nm when it scatters 90° from an electron.

10. Given

$$p = 9.1 \times 10^{-27} \text{ N} \cdot \text{s}$$

Required

wavelength (λ)

Analysis and Solution

$$\begin{aligned} p &= \frac{h}{\lambda} \\ \lambda &= \frac{h}{p} \\ &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{9.1 \times 10^{-27} \text{ N} \cdot \text{s}} \\ &= 7.3 \times 10^{-8} \text{ m} \\ &= 73 \text{ nm} \end{aligned}$$

Paraphrase

An electron with momentum $9.1 \times 10^{-27} \text{ N} \cdot \text{s}$ has a wavelength of 73 nm.

11. Given

$$\lambda = 100 \text{ nm} = 1.00 \times 10^{-7} \text{ m}$$

Required

momentum (p)

Analysis and Solution

$$\begin{aligned} p &= \frac{h}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{1.00 \times 10^{-7} \text{ m}} \\ &= 6.63 \times 10^{-27} \text{ N} \cdot \text{s} \end{aligned}$$

Paraphrase

A 100-nm UV photon has a momentum of $6.63 \times 10^{-27} \text{ N} \cdot \text{s}$.

- 12.** No, the particle cannot be at rest. Because it is confined to a small region, you know its position with a relatively small uncertainty. According to Heisenberg's uncertainty principle, however, there must be a corresponding very large uncertainty in the particle's momentum: The particle could be moving very fast and have considerable kinetic energy.

Applications

13. Given

$$P = 1.0 \text{ W} = 1.0 \text{ J/s}$$

$$\lambda = 600 \text{ nm} = 6.00 \times 10^{-7} \text{ m}$$

$$\Delta t = 1 \text{ s}$$

Required

number of photons (n)

Analysis and Solution

$$\begin{aligned}
 P &= \frac{E}{\Delta t} \\
 &= \frac{nhf}{\Delta t} \\
 &= \frac{nh\left(\frac{c}{\lambda}\right)}{\Delta t}
 \end{aligned}$$

For $\Delta t = 1 \text{ s}$,

$$\begin{aligned}
 P &= \frac{nhc}{\lambda} \\
 n &= \frac{P\lambda}{hc}
 \end{aligned}$$

The number of photons emitted each second is

$$\begin{aligned}
 n &= \frac{\left(1.0 \frac{\text{J}}{\text{s}}\right)\left(6.00 \times 10^{-7} \text{ m}\right)}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} \\
 &= 3.0 \times 10^{18} \text{ photons/s}
 \end{aligned}$$

Paraphrase

A 1.0-W flashlight emits 3.0×10^{18} photons of 600-nm light in one second.

14. Given

$$\lambda = 300 \text{ nm} = 3.00 \times 10^{-7} \text{ m}$$

$$W = 1.88 \text{ eV}$$

Required

maximum kinetic energy of the photoelectrons ($E_{k_{\max}}$)

Analysis and Solution

$$E_{k_{\max}} = hf - W$$

$$= \frac{hc}{\lambda} - W$$

$$= \frac{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)}{3.00 \times 10^{-7} \text{ m}} - (1.88 \text{ eV})\left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}}\right)$$

$$= 3.62 \times 10^{-19} \text{ J}$$

$$= \frac{3.62 \times 10^{-19} \text{ J}}{1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}}}$$

$$= 2.26 \text{ eV}$$

Paraphrase

The maximum energy of the electrons emitted by the metal surface is $3.62 \times 10^{-19} \text{ J}$ or 2.26 eV.

15. Given

$$V = 100 \text{ kV} = 1.00 \times 10^5 \text{ V}$$

Required

electron wavelength (λ)

Analysis and Solution

Use the equation $\lambda = \frac{h}{p}$. Relate p to E_k by noting that the electrons acquire kinetic

energy from work done by the accelerating electric field, defined by the equation

$$E_k = qV.$$

$$E_k = \frac{p^2}{2m}$$

$$p = \sqrt{2mE_k}$$

$$= \sqrt{2mqV}$$

Therefore,

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{\sqrt{2mqV}}$$

$$\begin{aligned}\lambda &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{\sqrt{2(9.11 \times 10^{-31} \text{ kg})(1.60 \times 10^{-19} \text{ C})(1.00 \times 10^5 \text{ V})}} \\ &= 3.88 \times 10^{-12} \text{ m} \\ &= 0.00388 \text{ nm}\end{aligned}$$

Paraphrase and Verify

The wavelength of a 100-keV electron in the microscope is 0.00388 nm, which is more than 100 000 times smaller than visible light. For this reason, an electron microscope has a much higher magnification, and is consequently capable of showing much greater detail, than a light microscope.

16. (a) Given

$$m = 0.15 \text{ kg}$$

$$v = 40 \text{ m/s}$$

Required

wavelength (λ)

Analysis and Solution

$$\lambda = \frac{h}{p}$$

$$= \frac{h}{mv}$$

$$= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(0.15 \text{ kg})(40 \text{ m/s})}$$

$$= 1.1 \times 10^{-34} \text{ m}$$

Paraphrase

The ball's wavelength is $1.1 \times 10^{-34} \text{ m}$.

- (b) You can safely ignore quantum effects because any uncertainty in the position of the ball due to quantum effects will be insignificant—the wavelength of the ball is only in the order of 10^{-34} m!

17. Given

$$d = 100 \text{ m}$$

$$P = 100 \text{ W} = 100 \text{ J/s}$$

$$d_{\text{pupil}} = 2 \text{ mm}$$

Required

number of photons entering your eye each second (n)

Analysis and Solution

$$\begin{aligned} P &= \frac{E}{\Delta t} \\ &= \frac{nhf}{\Delta t} \\ &= \frac{nh\left(\frac{c}{\lambda}\right)}{\Delta t} \end{aligned}$$

Estimate that the photons have an average wavelength of 500 nm. Then, determine the number of photons emitted each second.

For $\Delta t = 1 \text{ s}$,

$$\begin{aligned} P &= \frac{nhc}{\lambda} \\ n &= \frac{P\lambda}{hc} \\ &= \frac{\left(100 \frac{\text{J}}{\text{s}}\right)\left(5.00 \times 10^{-7} \text{ m}\right)}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right)\left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} \\ &= 2.51 \times 10^{20} \text{ photons/s} \end{aligned}$$

From a distance of 100 m, these photons will be spread over an area $A = 4\pi r^2$. So, the density of photons per square metre is

$$\begin{aligned} \frac{n}{4\pi r^2} &= \frac{2.51 \times 10^{20} \text{ photons/s}}{4\pi(100 \text{ m})^2} \\ &= 2.00 \times 10^{15} \text{ photons/s/m}^2 \end{aligned}$$

Your eye has an area of $\pi r^2 = \pi(1 \text{ mm})^2 = \pi(0.001 \text{ m})^2 = \pi \times 10^{-6} \text{ m}^2$. The number of photons entering your pupil each second is

$$\begin{aligned} \left(2.00 \times 10^{15} \text{ photons/s/m}^2\right)\left(\pi \times 10^{-6} \text{ m}^2\right) &= 2.00\pi \times 10^9 \text{ photons/s} \\ &= 6.28 \times 10^9 \text{ photons/s} \end{aligned}$$

Paraphrase

There are about six billion photons entering your eye each second.

18. Given

$$P = 200 \text{ kW} = 2.00 \times 10^5 \text{ W} = 2.00 \times 10^5 \text{ J/s}$$

$$f = 90.9 \text{ MHz} = 90.9 \times 10^6 \text{ Hz}$$

Required

number of photons emitted each second (n)

Analysis and Solution

$$P = \frac{E}{\Delta t}$$

$$= \frac{nhf}{\Delta t}$$

For $\Delta t = 1 \text{ s}$,

$$P = nhf$$

$$n = \frac{P}{hf}$$

$$n = \frac{2.00 \times 10^5 \frac{\text{J}}{\text{s}}}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (90.9 \times 10^6 \text{ s}^{-1})}$$

$$= 3.32 \times 10^{30} \text{ photons/s}$$

Paraphrase

The FM radio station emits 3.32×10^{30} photons each second.

19. Given

$$l = 0.85 \text{ nm} = 0.85 \times 10^{-9} \text{ m}$$

Required

three lowest possible energies for the electron (E_n)

Analysis and Solution

Use the equation $E_n = \frac{n^2 h^2}{8ml^2}$. Substitute $n = 1, 2, 3$, and solve. Alternatively, solve for

E_1 and then use $E_n = n^2 E_1$. The mass of an electron is $9.11 \times 10^{-31} \text{ kg}$.

$$E_1 = \frac{(1)^2 (6.63 \times 10^{-34} \text{ J} \cdot \text{s})^2}{8(9.11 \times 10^{-31} \text{ kg})(0.85 \times 10^{-9} \text{ m})^2}$$

$$= 8.3 \times 10^{-20} \text{ J}$$

$$= \frac{8.3 \times 10^{-20} \frac{\text{J}}{1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}}}}{1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}}}$$

$$= 0.52 \text{ eV}$$

$$E_2 = (2)^2 E_1 = 2.1 \text{ eV}$$

$$E_3 = (3)^2 E_1 = 4.7 \text{ eV}$$

Paraphrase and Verify

The lowest three energies for the particle in the 0.85-nm box are 0.52 eV, 2.1 eV, and 4.7 eV. The de Broglie wavelength for the particle must be a standing wave, so the only possible values of n are whole-number integers.

20. Given

$$E = 100 \text{ keV} = 1.00 \times 10^5 \text{ eV}$$

Required

momentum (p)

Analysis and Solution

Use the equations $E = hf = \frac{hc}{\lambda}$ and $p = \frac{h}{\lambda}$.

$$\begin{aligned}
 E &= \frac{hc}{\lambda} = \left(\frac{h}{\lambda}\right)c \\
 &= pc \\
 p &= \frac{E}{c} \\
 &= \frac{(1.00 \times 10^5 \text{ eV}) \left(1.60 \times 10^{-19} \frac{\text{J}}{\text{eV}}\right)}{3.00 \times 10^8 \text{ m/s}} \\
 &= 5.33 \times 10^{-23} \text{ kg} \cdot \text{m/s}
 \end{aligned}$$

Paraphrase

A 100-keV X-ray photon has a momentum of $5.33 \times 10^{-23} \text{ kg} \cdot \text{m/s}$.

Extensions

- 21.** Photons carry momentum. When a surface absorbs or reflects photons, the momentum of the photons changes. Because this change occurs over time, the photons must exert a force, such that $F = \frac{\Delta p}{\Delta t}$. If you calculate the average force over an area, then the photons must exert pressure, because $P = \frac{F}{A}$.

22. (a) Given

$$A = 1.0 \text{ km}^2 \quad F = 10 \text{ N} \quad E = 1.4 \text{ kW/m}^2$$

Required

number of photons per square metre (n)

Analysis and Solution

Choose 550 nm as the average wavelength of the photons.

$$\begin{aligned}
 P &= \frac{E}{\Delta t} = \frac{nh \left(\frac{c}{\lambda}\right)}{\Delta t} \\
 \text{For } \Delta t &= 1 \text{ s,} \\
 P &= \frac{nhc}{\lambda} \\
 n &= \frac{P\lambda}{hc} \\
 n &= \frac{\left(1.4 \times 10^3 \frac{\text{J}}{\text{s} \cdot \text{m}^2}\right) \left(5.50 \times 10^{-7} \text{ m}\right)}{\left(6.63 \times 10^{-34} \text{ J} \cdot \text{s}\right) \left(3.00 \times 10^8 \frac{\text{m}}{\text{s}}\right)} \\
 &= 3.87 \times 10^{21} \text{ photons/s/m}^2
 \end{aligned}$$

Paraphrase

The number of photons from the Sun is $3.87 \times 10^{21} \text{ photons/s/m}^2$.

(b) Given

$$A = 1.0 \text{ km}^2$$

$$F = 10 \text{ N}$$

Required

momentum of each photon (p)

Analysis and Solution

$$p = \frac{h}{\lambda}$$

Each photon has a wavelength of 550 nm and carries a momentum of

$$\begin{aligned} p &= \frac{h}{\lambda} \\ &= \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{550 \times 10^{-9} \text{ m}} \\ &= 1.21 \times 10^{-27} \text{ kg} \cdot \text{m/s} \end{aligned}$$

The photons are reflected, so

$$\begin{aligned} \Delta p &= 2p \\ &= 2(1.21 \times 10^{-27} \text{ kg} \cdot \text{m/s}) \\ &= 2.41 \times 10^{-27} \text{ kg} \cdot \text{m/s} \end{aligned}$$

Paraphrase

The momentum of each photon is $2.41 \times 10^{-27} \text{ kg} \cdot \text{m/s}$.

(c) Given

$$A = 1.0 \text{ km}^2$$

$$F = 10 \text{ N}$$

Required

the force that sunlight produces on the sail (F)

Analysis and Solution

From photon momentum, calculate the pressure the photons exert on the sail, per square metre.

$$\begin{aligned} P &= \frac{F}{A} \\ &= \frac{\Delta p}{A} \end{aligned}$$

For $\Delta t = 1 \text{ s}$,

$$P = \frac{\Delta p n}{A}, \text{ where } n \text{ is the number of photons per second calculated in (a).}$$

$$\begin{aligned} P &= \frac{(2.41 \times 10^{-27} \text{ kg} \cdot \text{m/s})(3.87 \times 10^{21})}{1 \text{ m}^2} \\ &= 9.33 \times 10^{-6} \text{ N/m}^2 \end{aligned}$$

Use your answer for pressure to calculate the force on the sail, using the equation

$$P = \frac{F}{A}.$$

$$\begin{aligned}
 F &= PA \\
 &= \left(9.33 \times 10^{-6} \frac{\text{N}}{\text{m}^2} \right) (1.0 \times 10^3 \text{ m})^2 \\
 &= 9.3 \text{ N}
 \end{aligned}$$

Paraphrase and Verify

The physicist's patent claim is valid! Your calculation shows that the sail gets a net push of 9.3 N from the Sun. This value is in good agreement with the claim that the thrust should be about 10 N.

23. Given

$$f = 100 \text{ MHz} = 1.00 \times 10^8 \text{ Hz}$$

$$I = 1.0 \text{ } \mu\text{A} = 1.0 \times 10^{-6} \text{ A}$$

$$V = 10 \text{ mV} = 1.0 \times 10^{-2} \text{ V}$$

Required

number of photons per second (n)

Analysis and Solution

Use the connection between electrical energy, current, and potential difference,

$$E = VI.$$

This energy comes from the photons interacting with the antenna.

$$E = nhf = VI$$

$$\begin{aligned}
 n &= \frac{VI}{hf} \\
 &= \frac{(1.0 \times 10^{-2} \text{ V})(1.0 \times 10^{-6} \text{ A})}{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(1.00 \times 10^8 \text{ s}^{-1})} \\
 &= \frac{1.0 \times 10^{-8} \cancel{\text{J}}}{6.63 \times 10^{-26} \cancel{\text{J}}} \\
 &= 1.5 \times 10^{17}
 \end{aligned}$$

Paraphrase

The minimum number of photons that your radio receiver can detect is in the order of 1.5×10^{17} .

- 24. (a)** Einstein was objecting to the central claim of Heisenberg's uncertainty principle, which states that you cannot know both the position and momentum of a particle at the same time with unlimited precision. Heisenberg's uncertainty principle implies that quantities such as position and momentum have a probabilistic nature. Einstein could not accept that, at the quantum level, we can, at best, determine the *probability* that a particle will be at a particular location or have a particular momentum.
- (b)** Einstein's statement is ironic because he was the one who suggested the quantum (particle) model of light—the first example of the wave-particle duality and one of the tenets of quantum indeterminacy.