Chapter 7 Review

Knowledge

1. Oscillatory motion is motion that repeats itself at regular intervals. For example, a mass oscillating on a spring and a pendulum swinging back and forth.

2. A ball will bounce with oscillatory motion if it has a uniform period. This means that it must bounce to the same height in each oscillation.

3. An elastic material will deform due to a stress. Once the stress is removed, the material will return to its original shape, without loss of energy.

4. The restoring force is the only force that acts on an isolated, frictionless, simple harmonic oscillator.

5. The direction of the restoring force is always opposite to the displacement.

6. The slope represents the spring constant.

7. It is not in its equilibrium position.

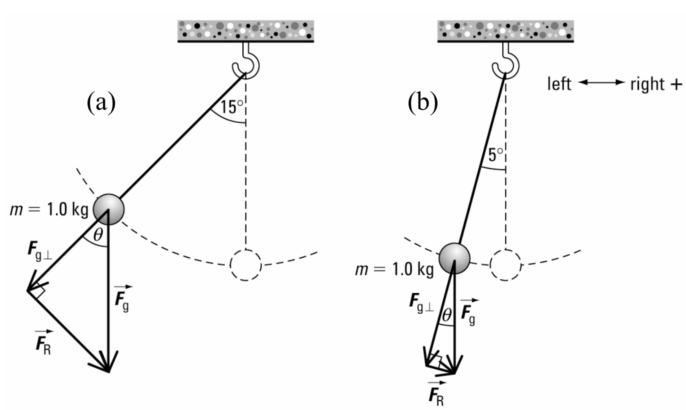
8. Acceleration depends on displacement, as given by $\vec{a} = \frac{-k\vec{x}}{m}$. Therefore, it is not uniform.

9. The restoring force varies with the sine of the angle of displacement. For small angles, this relationship is almost linear. As the displacement angle increases, the relationship is no longer linear. This difference begins to show at approximately 15°.

10. If the forced frequency is similar to the natural, resonant frequency, the amplitude of the oscillatory motion will be increased. The motion still takes place at the resonant frequency.

Applications

11. (a) ***Analysis and Solution***



Note that the angles in the diagram have been exaggerated for illustration purposes.

$$\begin{aligned} F_{g\perp} &= F_g \sin\ \theta \\ &= mg \sin\ \theta \\ &= (1.0\,\text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)\sin\ 15° \\ &= 2.5\,\text{N} \end{aligned}$$

The restoring force on the pendulum is 2.5 N [toward equilibrium].

(b) ***Analysis and Solution***

$$\begin{aligned} F_{g\perp} &= F_g \sin\ \theta \\ &= mg \sin\ \theta \\ &= (1.0\,\text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)\sin\ 5° \\ &= 0.85\,\text{N} \end{aligned}$$

The restoring force on the pendulum is 0.85 N [toward equilibrium].

12. ***Analysis and Solution***

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{0.0040\,\text{s}} \\ &= 2.5 \times 10^2\ \text{Hz} \end{aligned}$$

The frequency of the guitar string is 2.5×10^2 Hz.

13. ***Analysis and Solution***

$$\begin{aligned} T &= \frac{1}{f} \\ &= \frac{1}{0.67\ \text{Hz}} \\ &= 1.5\ \text{s} \end{aligned}$$

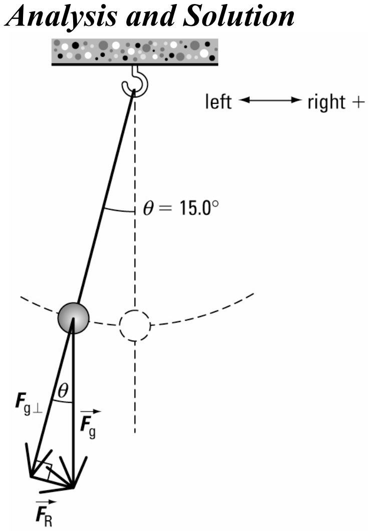
The period is 1.5 s.

14. ***Analysis and Solution***

$$\begin{aligned} f &= \frac{1}{T} \\ &= \frac{1}{0.100\ \text{s}} \\ &= 10.0\ \text{Hz} \end{aligned}$$

The frequency is 10.0 Hz.

15. ***Analysis and Solution***



$$\begin{aligned} F_{g\perp} &= F_g \sin\ \theta \\ &= mg \sin\ \theta \\ &= (2.0\,\text{kg})\left(9.81\frac{\text{m}}{\text{s}^2}\right)(\sin\ 15.0°) \\ &= 5.1\ \text{N} \end{aligned}$$

The restoring force acting on the pendulum is 5.1 N [toward equilibrium].

16. ***Analysis and Solution***

Any two points from the line of best fit can be used to determine the slope.
The following points were used for the solution below:
$(x_1, y_1) = (0.0, 0.0)$ and $(x_2, y_2) = (0.75, 150)$

$$\begin{aligned} k &= \text{slope} \\ &= \frac{(150 - 0.0)\ \text{N}}{(0.75 - 0.0)\ \text{m}} \\ &= 2.0 \times 10^2\ \text{N/m} \end{aligned}$$

The spring constant of the spring is 2.0×10^2 N/m.

17. ***Given***

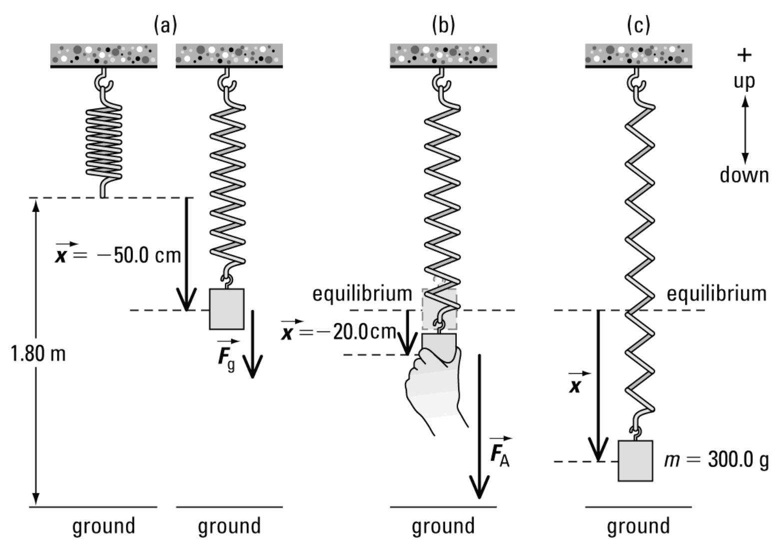
height of spring from floor = 1.80 m
height of spring from floor with 100.0-g mass attached =1.30 m

Required

(a) spring constant (k)
(b) force needed to pull the 100.0-g mass through a displacement of 20.0 cm ($\vec{F}$)
(c) distance of the 300.0-g mass ($\vec{x}$)

Analysis and Solution

Draw diagram of the situation described in each part the problem.



You need to know the spring constant of the spring to determine the force and displacements in parts (b) and (c). To determine the spring constant, use the force of gravity for the 100.0-g mass. The displacement of the mass-spring is 50.0 cm down.

(a)

$$\vec{x} = 0.500 \text{ m [down]}$$

$$\vec{F}_g = k\vec{x}$$

$$k = \frac{\vec{F}_g}{\vec{x}}$$

$$= \frac{mg}{\vec{x}}$$

$$= \frac{(0.1000 \text{ kg})\left(-9.81 \frac{\text{m}}{\text{s}^2}\right)}{-0.500 \text{ m}}$$

$$= 1.962 \text{ N/m}$$

$$= 1.96 \text{ N/m}$$

(b) Take the displacement of the 100.0-g mass (1.30 m) from the floor as the new equilibrium position, and determine the force required to pull it down 20.0 cm.

$$\vec{x} = 0.200 \text{ m [down]}$$

$$\vec{F} = k\vec{x}$$

$$= (1.962 \frac{\text{N}}{\text{m}})(-0.200 \text{ m})$$

$$= -0.392 \text{ N}$$

(c) Determine the distance for the 300.0-g mass from the spring's original equilibrium position (1.80 m) from the floor. Use the spring constant found in part (a) to determine this distance.

$$\vec{x} = \frac{\vec{F}_g}{k}$$

$$= \frac{mg}{k}$$

$$= \frac{(0.3000 \text{ kg})\left(-9.81 \frac{\text{m}}{\text{s}^2}\right)}{1.962 \frac{\text{N}}{\text{m}}}$$

$$= -1.50 \text{ m}$$

The height from the floor is:

$1.80 \text{ m} - 1.50 \text{ m} = 0.30 \text{ m}$

To three significant digits, the distance is 0.300 m.

Paraphrase

(a) The spring constant of the spring is 1.96 N/m.

(b) A person must apply a force of 0.392 N to pull the hanging 100.0-g mass down through a displacement of 20.0 cm.

(c) If the 100.0-g mass is removed and a 300.0-g mass is attached, it will hang 0.300 m from the floor.

18. ***Given***

$k_A = 100.0\text{ N/m}$

$k_B = 50.0\text{ N/m}$

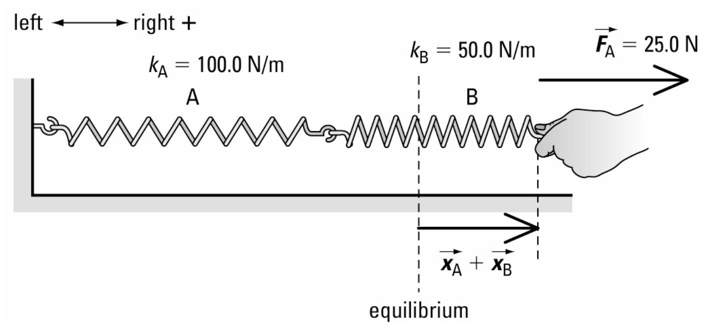
$\vec{F} = 25.0\text{ N [right]}$

Required

combined stretch of the two springs (x_T)

Analysis and Solution

Each spring will be stretched differently by the force. To determine the total stretch, calculate the stretch of each spring then add them up.



Spring A:

$$k_A = 100.0\text{ N/m}$$

$$\vec{F}_A = k\vec{x}_A$$

$$\begin{aligned}\vec{x}_A &= \frac{\vec{F}_A}{k_A}\\ &= \frac{25.0\ \cancel{\text{N}}}{100.0\frac{\cancel{\text{N}}}{\text{m}}}\\ &= 0.250\text{ m}\end{aligned}$$

Spring B:

$$k_B = 50.0\text{ N/m}$$

$$\vec{F}_B = k\vec{x}_B$$

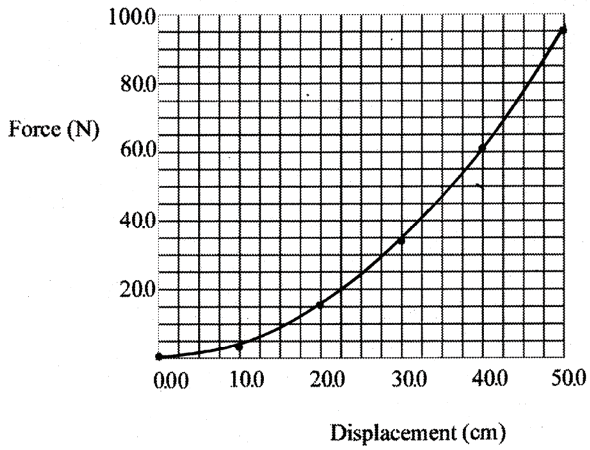
$$\begin{aligned}\vec{x}_B &= \frac{\vec{F}_B}{k_B}\\ &= \frac{25.0\ \cancel{\text{N}}}{50.0\frac{\cancel{\text{N}}}{\text{m}}}\\ &= 0.500\text{ m}\end{aligned}$$

$$\begin{aligned}\vec{x}_T &= \vec{x}_A + \vec{x}_B\\ &= 0.250\text{ m} + 0.500\text{ m}\\ &= 0.750\text{ m}\end{aligned}$$

Paraphrase
The combined stretch of the two springs is 0.750 m.

19.

Force vs. Displacement



The graph shows that the elastic band does not obey Hooke's law because the graph is not linear.

20. ***Analysis and Solution***

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$l = \frac{T^2 g}{4\pi^2}$$

$$= \frac{(1.00\,\text{s})^2\left(9.81\,\frac{\text{N}}{\text{kg}}\right)}{4\pi^2}$$

$$= \frac{(1.00\,\cancel{\text{s}})^2\left(9.81\,\frac{\text{m}}{\cancel{\text{s}^2}}\right)}{4\pi^2}$$

$$= 0.248\ \text{m or } 24.8\ \text{cm}$$

The length of the pendulum must be 24.8 cm.

21. ***Analysis and Solution***

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{10.0\ \text{kg}}{44.0\ \frac{\text{N}}{\text{m}}}}$$

$$= 3.00\ \text{s}$$

The period of the mass is 3.00 s.

22. ***Analysis and Solution***

$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$= 0.120\text{ m}\sqrt{\frac{2000.0\frac{\text{N}}{\text{m}}}{2.00\times10^{3}\text{ kg}}}$$

$$= 0.120\text{ m/s}$$

The maximum speed of the crate is 0.120 m/s.

23. ***Given***

A = 0.040 m

v_{max} = 0.100 m/s

m = 0.480 g = 0.000480 kg

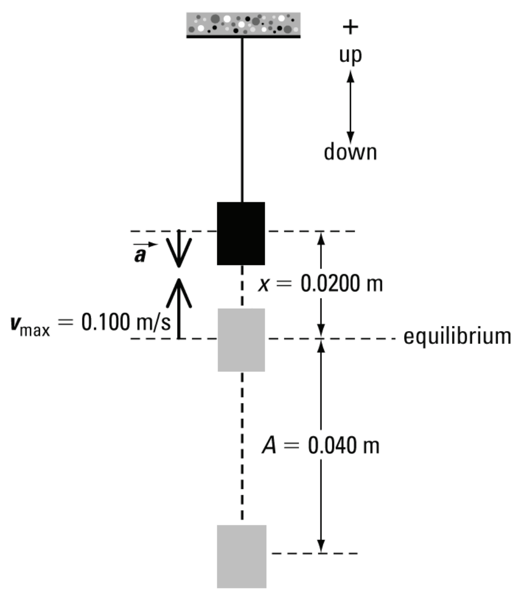
$\vec{x}$ = 0.0200 m [upward]

Required

acceleration ($\vec{a}$)

Analysis and Solution

To determine the acceleration of the mass, find the spring constant of its thread. Do this first and then use the spring constant in the acceleration equation. Make sure to use the appropriate SI units for the mass. Take up as positive, and down as negative.



$$v_{max} = A\sqrt{\frac{k}{m}}$$

$$v_{max}^2 = A^2\frac{k}{m}$$

$$k = \frac{v_{max}^2 m}{A^2}$$

$$= \frac{\left(0.100\frac{\text{m}}{\text{s}}\right)^2(4.80\times10^{-4}\text{ kg})}{(0.040\text{ m})^2}$$

$$= 3.0\times10^{-3}\text{ N/m}$$

Use this spring constant to determine the acceleration of the mass at a displacement of 0.0200 m [up].

$$\vec{F} = -k\vec{x}$$

$$m\vec{a} = -k\vec{x}$$

$$\vec{a} = \frac{-k\vec{x}}{m}$$

$$= \frac{-\left(3.0\times10^{-3}\frac{\text{N}}{\text{m}}\right)(0.0200\text{ m})}{4.80\times10^{-4}\text{ kg}}$$

$$= -0.13\text{ N/kg}$$

$$= -0.13\text{ m/s}^2$$

Paraphrase

The mass experiences an acceleration of 0.13 m/s^2 [down], when it is at a displacement of 0.0200 m [up].

24. ***Analysis and Solution***

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$= 2\pi\sqrt{\frac{0.2585\text{ m}}{\left(9.81\frac{\text{m}}{\text{s}^2}\right)}}$$

$$= 1.02\text{ s}$$

The period of the pendulum is 1.02 s.

25. ***Analysis and Solution***

$$T = \frac{1}{f}$$

$$= \frac{1}{0.182 \text{ Hz}}$$

$$= 5.49 \text{ s}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$= \frac{4\pi^2 0.50 \text{ m}}{(5.49 \text{ s})^2}$$

$$= 0.65 \text{ m/s}^2$$

$$= 0.65 \text{ N/kg}$$

Pluto's gravitational field strength is 0.65 N/kg.

26. (a) X represents $\frac{l}{g}$.

(b) ***Analysis and Solution***

$$X = \frac{l}{g}$$

$$\frac{l}{g} = \frac{T^2}{4\pi^2}$$

$$= \frac{(1.79 \text{ s})^2}{4\pi^2}$$

$$= 0.081\,16 \text{ s}^2$$

$$l = 0.081\,16 \text{ s}^2 (g)$$

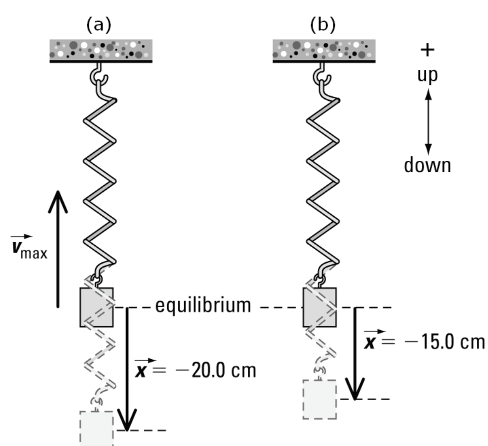
$$= 0.081\,16 \cancel{\text{s}^2}\left(9.81 \frac{\text{m}}{\cancel{\text{s}^2}}\right)$$

$$= 0.796 \text{ m}$$

The length of the pendulum is 0.796 m.

Extensions

27.



(a) ***Analysis and Solution***

$$v_{\max} = A\sqrt{\frac{k}{m}}$$

$$= 0.200 \text{ m}\sqrt{\frac{10.0 \ \frac{\text{N}}{\text{m}}}{0.2500 \text{ kg}}}$$

$$= 1.26 \text{ m/s}$$

The maximum speed of the mass is 1.26 m/s.

(b) ***Analysis and Solution***

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$= 2\pi\sqrt{\frac{0.2500 \text{ kg}}{10.0 \ \frac{\text{N}}{\text{m}}}}$$

$$= 0.993 \text{ s}$$

The mass will have a period of oscillation of 0.993 s. The period is independent of the amplitude.

28. (a) ***Given***

f = 0.800 Hz

Required

frequency of the oscillator when the mass is doubled (f)

Analysis and Solution

To find the frequency when the mass is doubled, modify the equation for the period to solve for frequency. Substitute 2m in place of m in the equation to determine the factor by which the frequency increases or decreases. Then determine the actual change to the frequency by multiplying this factor by the original frequency.

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$f = \frac{1}{T}$$

$$= \frac{1}{2\pi\sqrt{\frac{m}{k}}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Change m to $2m$.

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{2m}} \qquad \sqrt{2} = 1.414$$

$$= \frac{1}{2\pi(1.414)}\sqrt{\frac{k}{m}}$$

$$= \left(\frac{1}{1.414}\right)\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

$$= (0.707)\frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The frequency has changed by a factor of 0.707. Now determine the new frequency.

$$f = 0.707 \times 0.800 \text{ Hz}$$

$$= 0.566 \text{ Hz}$$

Paraphrase

The frequency of the oscillator will change from 0.800 Hz to 0.566 Hz.

(b) For any simple harmonic oscillator, the amplitude of oscillation does not affect the period, and therefore does not affect the frequency. The frequency will stay at 0.800 Hz.

29. (a) A bouncing ball is not a simple harmonic oscillator because the force does not vary with displacement. The force of gravity is constant regardless of the position of the ball.

(b) The movement of a puck on the ice cannot be considered SHM. The force applied by the stick is constant throughout its displacement.

(c) A plucked guitar string is an example of SHM. The guitar string's restoring force varies directly with its displacement.

